WHAT IS VENTILATION EFFICIENCY?

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Introduction

The main objective of a mechanical ventilation system can be defined as follows.

- To provide an adequate ventilation flow, with regard to cooling or heating requirements or the amount of pollution, for those parts of a room where it is required.

The realization of the main objective is affected by at least two constraints:

- The *comfort constraint.* Discomfort due to excessive air motion (draughts) must be avoided.
- The *financial constraint.* The investment and operating costs must be as low as possible.

The main objective together with its constraints raises the question of measuring the actual air distribution and defining a measure of the system's performance with regard to 'providing air in those parts in a room where it is required'.

The problem is illustrated in figure 1 , which shows a smoke visualization of a ventilation system in operation. Air is supplied through an air terminal mounted centrally in the ceiling and the exhaust air terminal is mounted in the wall just beneath the ceiling. The supply air has an overtemperature relative to the mean room temperature. The figure shows that only a small proportion of the supply air enters the occupied zone.

This merely shows qualitatively that this system is not efficient in 'providing air to those parts of a room where it is needed'. Assessing whether a particular ventilation system fulfils the main objective requires some means of measuring its effectiveness in terms of the dilution of air pollutants. Two methods are available, direct measurements of air flows and tracer gas dilution.

The detailed measurements of air movements is technically extremely difficult and complicated calculations are required (see equation 9) to determine the distribution of a pollutant in a room from such measurements. In contrast the use of a tracer gas is more attractive from a practical viewpoint. It also enables the direct observation of the system's capacity to 'wash out' pollutant. Measurements of local ventilation efficiency using the tracer gas technique have been carried out previously. Results from measurements in fullscale rooms are reported in [8,9] and in scale-models in reference [13]. The definition of local ventilation efficiency in references [8,14] is based on the slope of the tracer gas curves. But, as will be illustrated later, this method involves a number of difficulties. The definition in the reference [9] is based on the area under the curves. The use of radioisotope tracers to measure the ventilation efficiency is reported in [5].

The purpose of this article is threefold:

1. To consider, in the case of the tracer gas technique, the alternative definitions of a ventilation system's efficiency based on:

- *The slope of the tracer gas curves,*
- *The ratio between concentrations,*
- *The area under the curves.*

2. To study the connection between the transient and the steady states and to discuss what conclusions concerning steady-state behaviour can be drawn from 'transient analysis'.

3. To present measurements of ventilation efficiency which show that sometimes only a small proportion of the ventilation flow enters the occupied zone.

Definition of ventilation efficiency

The definition used should express the system's ability to remove pollution originating from a source in a room.

Figure 2 shows the changes of concentration in different parts of a room where pollution production is constant, i.e. independent of both time and space. At time $\tau = 0$, there was an even pollution concentration, $C(0)$, in the whole room. Two phases are involved, a *transient* and a *steady-state* phase.

To characterize fully a system its performance in both phases must be described. In flats and houses longer term pollution level is most important, but in industry the transient phase may also be important. For example, imagine a situation involving the sudden release of a toxic gas. In this type of case it may be important to know how rapidly the concentration can be brought below a certain level.

The definition or *ventilation efficiency at steady state* can be based on two characteristics of the ventilation system.

- *The relative ventilation efficiency,* which expresses how the system's ventilation ability varies between different parts of a room,

- *The absolute ventilation efficiency,* which expresses the ability of the ventilation system to reduce a pollution concentration in relation to the feasible theoretical maximum.

Using *values of the steady state condition, the relative ventilation efficiency,* ε_j^r, in a given point, j, in the room may be defined as:

$$\varepsilon_j^r = \frac{C_f^s - C_t}{C_j^s - C_t} \tag{1}$$

where C_f^s is the concentration in the exhaust air terminal, C_j^s is the concentration at point j and C_t is the concentration in the supply air terminal.
The value of the relative ventilation efficiency is always positive and can be greater than 1. When $C_j = C_f$ then $\varepsilon_j^r = 1$. The relative ventilation efficiency is a measure of dispersion and does not take into account absolute concentration levels or concentration changes from the initial concentration level.

An overall measure of the relative ventilation efficiency is provided by substituting the mean concentration in the whole room $\bar{C}$ in the definition above, for the local contamination concentration, C_j.

$$\bar{\varepsilon}^r = \frac{C_f^s - C_t}{\bar{C}^s - C_t} \tag{2}$$

(2) is the definition of the relative ventilation efficiency as given by Rydberg [11] and reflects the overall performance of the ventilation system. A transient relative efficiency in analogous to the definitions (1) and (2) can also be defined in the transient phase, this will be shown later in this article.

The *absolute ventilation efficiency*, ε_j^a, is *defined from values in the steady state* by:

$$\varepsilon_j^a = \frac{C(0) - C_j^s}{\Delta C_{max}} = \frac{C(0) - C_j^s}{C(0) - C_t} \qquad (3)$$

The absolute ventilation efficiency is always less than 1 and involves the 'direction' of change, see figure 3. Another motive for introducing the concept of absolute ventilation efficiency is that, as will be illustrated later, the relative ventilation efficiency can decrease when the ventilation flow increases. In this case absolute ventilation efficiency is a better measure of the true, or absolute, change in efficiency.

The above definitions primarily describe a ventilation system's performance over a 'long-term' period and therefore a poorer measure of its ability to remove 'transients'. An obvious way of characterizing this performance would be to take *the decay rate, i.e. slope* of individual curves.

Ventilation efficiency can be determined in two ways:

- *Direct measurement*, in which experiments are carried out to simulate the conditions under which the ventilation system is to operate either in the building or in a mock-up. The stationary pollution concentrations are quantified and inserted in definitions (1) to (3).

- *Indirect method,* in which attempts are made to identify quantifiable 'system paramenters' which characterize the system's ability to evacuate pollution. From these parameters it is possible to calculate the ventilation efficiency, as defined in (1) to (3).

A crucial question is; Is there any alternative to the direct method? This would require system parameters to exist that fulfil the following criteria:

- They are general, in the sense that by using them system performance can be assessed for different operating conditions.

- They can be measured with less effort than is required to simulate the actual operating conditions.

The first criteria is important for the practising engineer. The system parameters are of no interest in themselves if the concentration cannot be correctely estimated from them.

System parameters

When the mixing is perfect the course of concentration changes in the transient phase and concentration levels at steady state are completely described by a single system parameter, the *nominal air-exchange rate* n:

$$n = \frac{Q}{V_r} \tag{4}$$

Where Q is the ventilation air flow supply, in units of volume per unit time, and V_r is the total room volume. In a system in which the air is not 'perfectly mixed', the air entering any part of the room is a mixture of recirculated and 'fresh' air and the system's behaviour is then more difficult to describe. The 'freshness' of the air and its dilution capability at a particular point is characterized by its 'age'. The age of the air at the point is defined as the time τ, that has elapsed since the air entered the room.

The statistical age-distribution at any point can be obtained experimentally by injecting a pulse of tracer gas in the inlet air duct and continously recording the concentration C_j at the point in question. The *age distribution* $A_j(\tau)$, at any point is then equal to:

$$A_j(\tau) = \frac{C_j(\tau)}{\int_0^\infty C_j(\tau)d\tau} \tag{5}$$

The *average age* or the *local residence time*, θ_j of the air at an arbitrary point is then defined as:

$$\theta_j \equiv \int_0^\infty A_j(\tau)\tau \, d\tau = \frac{\int_0^\infty C_j \tau \, d\tau}{\int_0^\infty C_j \, d\tau} \tag{6}$$

The 'younger' the air the better is its dilution capability. From the age-distribution any statistical parameter as e.g. the variance and the skewness can be derived.

A mean recycle-time for the system can be defined from the age distribution in the exhaust air terminal, see [2].

A measure of the *local ventilation rate* r_j can be defined from (6) as:

$$r_j \equiv \frac{1}{\theta_j} \tag{7}$$

The local ventilation rate is a measure of the exchange of input air or 'fresh' air at the actual point. The definition (5), (6) and (7) are those normally found in chemical engineering, see e.g. [3].

A local air-exchange rate can be defined as the flow rate of air Q_j entering a certain volume V_j, irrespectively of the time the air has been in the system. The *local air-exchange rate* n_j is defined as:

$$n_j = \frac{Q_j}{V_j} \tag{8}$$

The local air-exchange rate is always equal to or higher than the local ventilation rate. When the mixing is complete the local air exchange rate is equal to the nominal air exchange rate. The local air-exchange rate is a more fundamental physical measure, than a practical ventilation performance measure, and is therefore not so interesting in this context.

The system's parameters are estimated from observing the transient behaviour of the concentration at different points. Three transient methods will be discussed briefly in the next section.

Transient experimental methods

Method 1. The 'decay' method

- The room is filled with tracer gas. With the aid of fans the gas is mixed to an even initial concentration C_o.
- The fans are turned off and the decay of the tracer gas is continuously recorded at different points.

Method 2. The 'source' method

- A constant flow of tracer gas is admitted to the supply air duct, i.e. the concentration in the supply air C_t is held constant.
- The growth of the tracer gas concentration is continously recorded at different points.

Method 3. The 'pulse' method

- A small tracer gas quantity m is admitted to the supply air duct.
- The growth and decay of the tracer gas concentrations is continously recorded at different points.

The transient behaviour of methods 1 and 2 are theoretically the same, if the velocity fields are equal. This is easily seen by defining the 'complementary transient concentration' $C^{tr} \equiv C_t - C^{tr}(\tau)$ in method 2. The transient behaviour of the 'complementary concentration' with the initial concentration $C_o \equiv C_t$, coincides with the concentration course in method 1. This means that conclusions about transient behaviour are equivalent for both methods, therefore from now on only method 2 will be considered with the transient concentration defined as above.

In *practise,* however, there is a difference between the first two methods. When method 1 is applied at the beginning of the measurement the velocity field created by the ventilation system is affected by the mixing-fans.

The equation of conservation

Various forms of the equation of conservation are needed at a later stage for the evaluation of different ways of defining and measuring ventilation efficiency.

It will be supposed that the pollution (tracer) is transported by convection and diffusion (turbulent and laminar). Its concentration C at a given point (x, y, z) in the room obeys the equation:

$$\delta C/\delta\tau = -\nabla(\underline{u}C) + \nabla(D\nabla C) + \dot{m} \qquad (9)$$

$\underline{u}$ (x, y, z) is the local convective velocity vector

D (x, y, z) is the local diffusivity

$\dot{m}$ (x, y, z) is the local pollution strength

$\nabla = (\frac{\delta}{\delta x}, \frac{\delta}{\delta y}, \frac{\delta}{\delta z})$ is the nabla operator

The velocity, diffusivity and the pollution production vary with position, but not with time and concentration.

The flow is incompressible, i.e.

$$\nabla \cdot \underline{u} = 0 \qquad (10)$$

The steady-state ($\delta C/\delta t = 0$) concentration C^s obeys the equation:

$$- \nabla(\underline{u}C^s) + \nabla(D\nabla C^s) + \dot{m}\ (x, y, z) = 0 \tag{11}$$

It follows from (11) that in the ideal case of zero diffusivity ($D = 0$) the steady state concentration would have been constant in the parts of the room with no pollution production ($\dot{m} = 0$). In the real situation with finite diffusivity there are also gradients where there is no pollution production and pollution may even be transporter in the opposite direction to the mean convective flow.

Integrating (11) over the whole room volume, assuming that the pollution concentration in the supply air terminal C_t is equal to zero, and that there are no gradients of C at the room walls (no leakage), the following expression is obtained for the pollution concentration, C_f^s, int the exhaust air terminal:

$$C_f^s = \frac{\iiint \dot{m}\ dx\,dy\,dz}{Q} \tag{12a}$$

With a *uniform pollution production* $\dot{m}$ in the whole room equation (11) can be divided by $\dot{m}$, giving

$$\nabla(\underline{u}\ \frac{C^s}{\dot{m}} - \nabla D\ \frac{C^s}{\dot{m}}) = 1 \tag{13}$$

and (12a) becomes

$$C_f^s = \frac{\dot{m} \cdot V_r}{Q} \tag{12b}$$

The expressions 12a and 12b are obvious because all pollution must pass the exhaust air terminal.

For the transient case, where $\dot{m} \equiv 0$, the concentration $C^{tr}(x, y, z)$ obeys the equation:

$$\delta C^{tr}/\delta\tau = - \nabla(\underline{u}C^{tr}) + \nabla(D\nabla C^{tr}) \tag{14}$$

Integration of equation (14) from $\tau = 0$ to $\tau = \infty$ and (as previously) assuming that $C_t = 0$ ($\Rightarrow C_\infty(x, y, z) = 0$), (14) becomes:

$$-C_o(x, y, z) = -\nabla(\underline{u} \int_0^\infty C^{tr}\, d\tau) + \nabla(D\nabla \int_0^\infty C^{tr}\, d\tau) \tag{15}$$

Where C_o (x, y, z) is the concentration at $\tau = 0$.

Assuming that *the conditions for the transient method 2* hold (i.e. the initial concentration C_o is uniform) both sides in equation (15) can be divided by C_o, obtaining:

$$\nabla(\underline{u} \frac{\int_0^\infty C^{tr}\, d\tau}{C_o} - D\nabla \frac{\int_0^\infty C^{tr}\, d\tau}{C_o}) = 1 \tag{16}$$

To avoid formal difficultied it is henceforth assumed that no pollution is transported by diffusion into the inlet air terminal against the convective flow. Integrating over the whole room volume (15) gives:

$$\iiint C_o(x, y, z)\, dx\, dy\, dz = Q \int C_f^{tr}\, d\tau \tag{17a}$$

With a uniform initial concentration, C_o, this becomes:

$$C_o V_r = Q \int_0^\infty C_f^{tr}\, d\tau \tag{17b}$$

These two last expressions are again trivial because they merely express that all the pollution must pass the exhaust air terminal.

When *the conditions for the transient method 3* are applied ($C_o(x, y, z) = C_\infty(x, y, z) \equiv 0$) it follows from (15), see Spalding [14], that *for each point in the room* the area under the curves is the same;

$$\int_0^\infty C^{tr}\, d\tau = \frac{m}{Q} \equiv \text{constant} \tag{18}$$

Where m is the amount of pollutant injected into the inlet-air duct.

To obtain an expression for the residence-time θ, the equation (14) is multiplied by the time τ and then integrated from $\tau = o$ to $\tau = \infty$, assuming that the conditions for the transient method 3 holds. Thus,

$$-\int_0^\infty C^{tr}\, d\tau = -\nabla\,(\underline{u}\int_0^\infty C^{tr}\tau d\tau) + \nabla\,(D\nabla\int_0^\infty C^{tr}\tau d\tau) \tag{19}$$

According to (18) the left-hand side (LHS) in (19) is constant, and therefore the whole equation can be divided by its LHS, obtaining the equation for the residence-time θ:

$$\nabla(\underline{u}\theta - D\nabla\theta) = 1 \tag{20}$$

Because it is assumed that there is no diffussion against the convective flow in the inlet-air duct, the residence- time in a point upstream of the injection point is zero. Therefore integrating (20) over the whole room volume gives, for the residence-time in the exhaust-air duct:

$$\theta_f = \frac{V_r}{Q} \tag{21a}$$

and for the ventilation rate in the exhaust-air duct:

$$r_f = \frac{Q}{V_r} \equiv n \tag{21b}$$

Finally with the same assumptions as above an equation for the *transient mean concentration* $\bar{C}^{tr}$ is obtained by integrating (14) over the whole room volume.

$$V_r \frac{d\bar{c}^{tr}}{d\tau} = - Q \, C_f^{tr} \tag{22}$$

Model cases

To determine the possibility of using the slope of the tracer gas decay curves and the ratio between the curves, three simple model cases will be discussed in some detail.

The room is considered as divided into two equal control volumes with volume V (see figure 4) where the lower volume represents the zone of occupance. In each zone the mixing is assumed complete, whereas the mixing between the two volumes is expressed by a parameter β that may vary from no mixing ($\beta = 0$) to complete mixing ($\beta \to \infty$). This parameter is a purely mathematical concept and the 'flow' βQ is not a physically measureable quantity, the parameter β is merely a measure of the degree of mixing.

Case a. The diplacement system

In this system the supply- and exhaust air terminals are placed far apart and the ventilation air is forced to pass across the room.

Under the assumption that the pollutant concentration C_t in the supply air is zero, the mass balance gives the following differential equation for the concentrations C_f and C_j.

$$V \frac{dC_f}{d\tau} = - (1 + \beta) QC_f + (1 + \beta) QC_j + \dot{m}_f$$

(23)

$$V \frac{dC_j}{d\tau} = \beta Q C_f - (1 + \beta) Q C_j + \dot{m}_j$$

where $\dot{m}_f$ and $\dot{m}_j$ represent the rate of pollution generation in the upper and lower control volumes.

Case b. The completely mixed system

In this case the concentration in every point in the room is the same and is described by the familiar equation:

$$V_r \frac{dC}{dt} = - Q C + \dot{m} \qquad (24)$$

Case c. The short cicuiting system

In this system there is a risk of short circuiting between the inlet- and the exhaust air terminals usually because they are close to each other. In terms of this simple model this means that they are directly connected to the same zone of mixing. The mass balance equations are therefore

$$V \frac{dC_f}{d\tau} = -(1 + \beta) Q C_f + \beta Q C_j + \dot{m}_f$$

$$V \frac{dC_j}{d\tau} = \beta Q C_f - \beta Q C_j + \dot{m}_j \qquad (25)$$

The local ventilation rate r for the systens may be obtained by applying the definitions from equations (6) and (7) and using equation (21b). The local ventilation rate and the local exchange rate are presented in table 1.

Table 1.

The local ventilation rates r_f, r_j and the local exchange rates n_f, n_j in the two control volumes, each with the volume V. Systems a, b and c as defined in text.

System	Local ventilation rate r_f r_j	Local air-exchange rate n_f n_j
a	$\frac{Q}{2V}$	$(1+\beta)\cdot\frac{Q}{V}$
	$\frac{(1+\beta)}{(1+2\beta)}\cdot\frac{Q}{V}$	$(1+\beta)\cdot\frac{Q}{V}$
b	$\frac{Q}{2V}$	∞
c	$\frac{Q}{2V}$	$(1+\beta)\cdot\frac{Q}{V}$
	$\frac{\beta}{(1+2\beta)} \quad \frac{Q}{V}$	$\beta\,\frac{Q}{V}$

The solutions of the equations (23) to (25) can be written in terms of a transient and a stationary part C^s.

$$C_f(\tau) = K_1 a_{11} e^{-n\sigma_1\tau} + K_2 a_{12} e^{-n\sigma_2\tau} + C_f^s$$

$$C_j(\tau) = K_1 a_{21} e^{-n\sigma_1\tau} + K_2 a_{22} e^{-n\sigma_2\tau} + C_j^s \tag{26}$$

Where

K_1, K_2	Constants depending on the initial conditions
$\left.\begin{array}{l}(a_{11}, a_{21})\\(a_{12}, a_{22})\end{array}\right\}$	Eigenvectors
$n\sigma_1$, $n\sigma_2$	Eigenvalues
$n = \frac{Q}{V_r}$	The nominal air-exchange rate

The eigenvalues for the different systems are listed in table 2 and the stationary pollution concentrations in the occupied zone C_j^s are given for different locations of the pollution source in table 3.

Table 2. The eigenvalues $n\sigma_1$ and $n\sigma_2$ for $\beta > 0$.

System	Eigenvalues $n\sigma_1$, $n\sigma_2$
a	$n\,(-2(1+\beta) \pm \sqrt{4\beta(\beta+1)}$
b	n, 0
c	$n\,(-(2\beta+1) \pm \sqrt{1+4\beta^2})$

Table 3. The stationary pollution concentration C_j^s in the zone of occupance for different locations of the pollution source $\dot{m}$. V_r is the total room volume.

The stationary pollution concentration C_f^s in the upper zone is according to (11b) always equal to $\frac{\dot{m}V_r}{Q}$

The corresponding relative ventilation efficiencies are listed in table 3.

Table 4. The stationary relative ventilation efficiency ε_j^s for different locations of the pollution source.

Tables 3 and 4 show that for a given pollution source, system a gives a lower stationary concentration level than system c. For both systems a and c the concentration becomes uniform in the whole room when the pollution source is located in the same mixing zone as the inlet air terminal. Furthermore, *complete mixing* is *the best feasible operative mode for system c* but *the poorest for system a*. This is best illustrated in figures 5a, 5b and 6. In the transient phase system a approaches the steady state concentration faster than the other two systems.

Transient analysis

The slope of the curves as a measure of ventilation efficiency

One natural approach would be, in analogy with infiltration measurements, to measure the decay rate, i.e. the slope, of the tracer gas curves at different points in a room. The evolution of the transient gas concentration C^{tr} in the model cases is shown in a linear logarithmic diagram in figures 5a and 5b.

In the first figure, 5a, which shows the system a, there is no mixing between the zones ($\beta = 0$). The decay rate in the lower volume is constant and equal to the exchange rate of the volume. The decay rate in the upper zone varies initially but approaches the decay rate in the lower zone and they become almost equal.

In all the other cases *there is exchange between the two control volumes* ($\beta \neq 0$) and it appears from the figures that *the decay rates* differ initially but that after a certain period of time, τ_0, they *become constant and equal.*

This behaviour is explained by examining the eigenvalues in table 1, where it appears that

$$|\sigma_1| < |\sigma_2|$$

This means that after a sufficient time has elapsed ($\tau \geq \tau_0$), the second term in the system of equations can be neglected, i.e. the gas concentrations are given by

$$\left.\begin{array}{l} C_f^{tr}(\tau) \sim K_1\, a_{11}\, e^{-n\sigma_1\tau} \\ C_j^{tr}(\tau) \sim K_1\, a_{21}\, e^{-n\sigma_1\tau} \end{array}\right\} \qquad \tau \geq \tau_0 \tag{27}$$

This implies that the ratio between the concentrations is constant i.e.

$$\frac{C_f^{tr}(\tau)}{C_j^{tr}(\tau)} \sim \frac{a_{11}}{a_{21}} = \hat{\varepsilon}_j^r \qquad \tau \geq \tau_0 \tag{28}$$

The constant ration $\hat{C}_i^r$ for the model cases is given in the table 5. Because the concentrations at each point decay at the same rate it also follows that the mean concentration in the room $\bar{C}$ decays at the same rate. In this case the ratio between the concentration in the exhaust air terminal and mean concentration is constant, i.e.

$$\frac{C_f^{tr}}{\bar{C}^{tr}} \equiv \hat{\varepsilon} \qquad \tau \geq \tau_0 \tag{29}$$

This relation inserted in (21) gives

$$\frac{d\bar{c}}{d\tau} = -n\, \hat{\varepsilon}\, \bar{c} \qquad \tau \geq \tau_0 \tag{30}$$

The solution of (29) is

$$\bar{C}(\tau) = \bar{C}(\tau_0)\, e^{-n\,\hat{\varepsilon}\tau} \tag{31}$$

and the slope in a linear logarithmic plot is equal to

$$n \hat{\bar{\varepsilon}} = n \frac{C_f^{tr}}{\bar{C}^{tr}} \qquad (32)$$

This has been pointed out earlier by Malmström [10]. For system a it holds that $\hat{\bar{\varepsilon}} \geq 1$ and for system c it holds that $\hat{\bar{\varepsilon}} \leq 1$. For an incompletely mixed system the above results can be summarized as follows:

- the decay rate varies initially both with location and time, but after a certain period of time it becomes constant and equal in different locations,
- when the decay rate is constant and equal at every point it reflects only the overall ventilation rate,
- no unique 'local air change rate' can be defined for the whole process from the slope of the tracer gas curves,
- it takes a certain period before the differences in ventilation efficiencies have completely taken effect.

One natural approach to characterizing the overall decay rate when mixing is not complete, would be to fit, to the measurement data, an expression of the type

$$C(\tau) \approx C(o)\ e^{-\gamma n \tau} \qquad (33)$$

The parameter γ is commonly referred to as the mixing factor in the USA [1, 4, 6, 7].

The value of this factor obtained is, of course, dependent on the period during which measurements have been made, and is therefore not an unequivocal measure. The longer the measuring time, the closer to $\hat{\varepsilon}$ will be the value of $\dot{\gamma}$ obtained. Nevertheless, when the deviation from complete mixing is dominant it is presumably a satisfactory measure of the overall ventilation rate.

It is not advisable, however to use an expression such as (33) to define the local ventilation rate at different points. The reason for this is twofold:

- Differences in local ventilation capability will be underestimated.
- As the measuring time approaches infinity the values will converge, i.e. there is a lack of consistency in the definition.

The two measures discussed next are more suitable for expressing local defferences.

The ratio between concentrations as a measure of ventilation efficiency

What is still left as a reminder of different local ventilation rates, when the decay rate is equal everywhere, are *the different concentrations* levels in different parts of the room. The ratio $\hat{\varepsilon}_j^r$, see (28), between the concentration in the exhaust air and in a given part of the room may be used as a measure of *'transient relative ventilation efficiency'*.

Table 5. The transient relative ventilation efficiency $\hat{\varepsilon}^r_j$.

System	$\hat{\varepsilon}^r_j$
a	$\sqrt{\frac{(1+\beta)}{\beta}}$
b	1
c	$\frac{2\beta}{1+\sqrt{1+4\beta^2}}$

A question that immediately arises here concerns the connection between the 'transient relative ventilation efficiency', $\hat{\varepsilon}^r_j$, and the relative ventilation efficiency ε^r_j as defined at steady state (1).

A guide to the answer is evident in figure 6, which shows the relative transient ventialtion efficiency and the stationary relative efficiency for different locations of the pollution source. Note that the transient and the stationary relative ventilation efficiencies *never* coincide, except in the ideal situation of compete mixing. The correspondence is best when the pollution source is homogenous, but is poorer for a non-homogeneous source. Nevertheless, the transient relative efficiency is proportional to the stationary one.

The transient relative ventilation efficiency can thus be used as an *estimate* of the relative ventilation efficiency at steady state. In other words, the transient relative ventilation efficiency is a *ventilation efficiency indicator or -index* of the ventilation efficiency at steady state.

The areas under the curves as a measure of ventilation efficiency

A transient ventilation efficiency may also be defined from the areas under the dilution curves as:

$$\hat{\varepsilon}_j^{r} \equiv \frac{\int_0^\infty C_f^{tr} d\tau}{\int_0^\infty C_j^{tr} d\tau} \qquad (34)$$

where C_j is the concentration at an arbitrary point in the room.

This measure takes into account all information in the dilution process. The connection between the transent relative ventilation efficiency as defined by (34), the residence-time and the stationary efficiency (1) will first be studied by comparing three cases. They are:

- A stationary case with s homogenous and passive pollution source $\dot{m}$.
- A transient case, where the transient method 2 is applied, starting from a homogeneous initial concentration C_o.
- A transient case where the transient method 3 is applied.

A "passive" pollution source is defined as an ideal source that does not affect the velocity field created by the ventilation system. The three cases are governed by the equations (13),(16) and (20).

$$\nabla \left(\underline{u}\,\frac{C_j^s}{\dot{m}} - D\nabla \frac{C_j^s}{\dot{m}}\right) = 1 \tag{13}$$

$$\nabla\left(\underline{u}\,\frac{\int_0^\infty C_j^{tr}d\tau}{C_o} - D\,\nabla\,\frac{\int_0^\infty C_j^{tr}d\tau}{C_o}\right) = 1 \tag{16}$$

$$\nabla\left(\underline{u}\,\theta_j - D\nabla\theta_j\right) = 1 \tag{20}$$

The above equations are in a dimensionless form. If the velocity field $\underline{u}$ the diffusivity D and the boundary conditions are the same in the equations above, they are mathematically identical and therefore have the same solution $F(\frac{\underline{x}.\underline{u}}{D})$ in an arbitrary point j, i.e.

$$\frac{C_j^s}{\dot{m}} = \frac{\int_0^\infty C_j^{tr}d\tau}{C_o} = \theta_j \equiv F\left(\frac{\underline{x}.\underline{u}}{D}\right) \tag{35}$$

The relation (35) can be written as:

$$C_j^s = \dot{m}\theta_j = \frac{\dot{m}}{r_j} \qquad r_j = \frac{C_o}{\int_0^\infty C_j^{tr}\,d\tau} \tag{36}$$

The preceding show that the local residence-time θ_j and the local ventilation rate r_j can be obtained by applying the transient method 2. Further, (36) shows that the "system parameters" θ_j and r_j can be used for calculation of the stationary concentrations, when the pollution source is homogeneous. After inserting (12b) respectively (17b) in (35) we obtain:

$$\varepsilon_j^r \equiv \frac{C_f^s}{C_j^s} = \frac{\int_0^\infty C_f^{tr}\,d\tau}{\int_0^\infty C_j^{tr}\,d\tau} \equiv \hat{\varepsilon}_j^r \qquad (37)$$

Equations (35) to (37) show that the stationary efficiency (relative and absolute) can be obtained from transient analysis. This is illustrated for the model cases in figur 6, where the transient efficiency $\hat{\varepsilon}_j^r$ coincide with the stationary efficiency ${}_0^0\varepsilon_j^r$ (uniform pollution source in the whole room). This conclusion is not only limited to hold for a homogeneous pollution source. It can easily be generalized to hold for an arbitrary passive pollution source distribution $\dot{m}$ (x, y, z). The condition will generally be that the initial concentration distribution is 'geometrically similar' to the pollution source distribution:

$$C_o\,(x, y, z) = \alpha\,\dot{m}\,(x, y, z) \qquad (38)$$

Where α is a dimensionally dependent proportionality factor. The stationary concentration now becomes:

$$C_j^s = \frac{\int_0^\infty C_j^{tr}\,d\tau}{\alpha} \qquad (39)$$

It is important to note that the integral in (39) is not equal to the integrals in (35) to (36), because the integral in (39) is obtained starting from a non-uniform initial concentration. From this it follows that in the general case there is no longer a simple relation between the stationary concentrations and the "system parameters" θ_j and r_j.

Finally it appears from figure 6 that the following relations hold between the transient relative efficiency defined as a ratio between concentrations $\hat{\varepsilon}_j^r$ and defined as a ratio between areas $\hat{\hat{\varepsilon}}_j^r$:

Case a ('The displacement system'): $\hat{\hat{\varepsilon}}_j^r \leq \hat{\varepsilon}_j^r$ (40)

Case c ('The short circuiting system'): $\hat{\hat{\varepsilon}}_j^r \geq \hat{\varepsilon}_j^r$ (41)

From the shape of the dilution curves for the different cases, the realtions (40) and (41) can be proven to hold generally.

Measurements

Measurements have been carried out in a room measuring (width x length x height) = 3.6 x 4.2 x 2.7 m. The supply air terminal was mounted centrally in the ceiling (see figure 1). The exhaust air terminal was placed above the door and 0.2 m beneth the ceiling. Due to the placing at the supply- and exhaust air terminals, this system can be classified as a short-circuiting system. A numbers of sensors were placed in the room at levels 0.3, 0.9 and 1.2 m above the floor, and one was placed in the exhaust air duct. The tracer gas used was (N_2O) and it's concentration was measured at these levels. The supply air flow was varied to correspond to 1, 2 and 4 nominal air changes. The supply air temperature was also varied to obtain overtemperatures of 0.2, 4 and 8°C above the mean temperature in the occupied zone (the zone up to 1.8 m above the floor).

Examples of results obtained by transient method 1 and 2 are found in table 6. The results obtained by method 2 are also shown in figure 7.

It is seen from table 6 that the values obtained by method 1 and 2 respecively, do not differ much. The only exception is at isothermal conditions with a low nominal air change rate. But the discrepancy between obtained values in this case probably do not depend so much on the applied methods themselves, as the flow pattern in the room then is more or less unstable. Due to this unstability the differences in obtained values probably reflect different flow patterns at the different measuring occasions.

Measured values of the transient relative ventilatio efficiency $\hat{\varepsilon}_j^r$ at different levels, j. The room height is 2.7 m.

Overtemperature	0°C						2°C					
Nominal airexchange rate	1(h^{-1})		2(h^{-1})		4(h^{-1})		1(h^{-1})		2(h^{-1})		4(h^{-1})	
Method / Level	1	2	1	2	1	2	1	2	1	2	1	2
0.3 m	0.62	0.85	0.92	0.92	0.96	1	0.70	0.68	0.65	0.66	0.75	0.88
0.9 m	0.78	0.94	0.92	1	0.95	1	0.66	0.66	0.61	0.63	0.71	0.83
1.5 m	0.92	0.97	0.93	1	0.94	0.99	0.66	0.65	0.62	0.62	0.77	0.89
2.1 m	1	0.98	0.92	1	0.93	0.98	0.78	0.76	0.83	0.84	0.90	1

Overtemperature	4°C						8°C					
Nominal airexchange rate	1(h^{-1})		2(h^{-1})		4(h^{-1})		1(h^{-1})		2(h^{-1})		4(h^{-1})	
Method / Level	1	2	1	2	1	2	1	2	1	2	1	2
0.3 m	0.58	0.60	0.52	0.57	0.69	0.87	0.55	0.56	0.47	0.46	0.50	0.47
0.9 m	0.56	0.59	0.50	0.51	0.57	0.83	0.53	0.54	0.45	0.45	0.43	0.44
1.5 m	0.55	0.56	0.48	0.49	0.76	0.82	0.51	0.53	0.44	0.48	0.43	0.43
2.1 m	0.62	0.63	0.73	0.75	0.91	1	0.54	0.55	0.56	0.57	0.88	0.99

Table 6 and figure 7 show the heavy 'layering effect' with an increase in the supply air temperature. The exchange of air between the upper part of the room and the zone of occupance is partly inhibited. The increase in the nominal air change rate, affects the ventilation efficiency in a perhaps somewhat unexpected manner. In the zone of occupance the relative ventilation efficiency *decreases* at non-isothermal conditions when the nominal air-exchange increases from 1 to 2 airchanges per hour. This behaviour has been observed before, see ref [12].

From the above results we may conclude that a ventilation system may be very inefficient in 'provaiding air where it is needed'.

The experience gained from measurements has also shown that the area of the dilution curve is a better defined measure than either the slope of the curve or the ratio between curves. The last two measure are less precise due to random fluctuations and to measuring errors in the data. Therefore, the ratio between the concentrations never becomes exactly constant and "smoothing" is needed to obtain the slope. The area provide a 'smoothed' measure.

CONCLUSIONS

Measurements show that large variations in the local ventilation efficiency can occur and should therefore be taken into account when a ventilation system is designed.

To be fully characterized a system's performance must be described in both the transient - and steady states.

The transient phase

In an incompletely mixed system a certain minimum period of time is required before the local differences in ventilation capability have completely taken effect.

A measure of the local ventilation efficiency can be based on:

- The slopes of the curves, i.e. the decay rates.
- The ratio between concentrations.
- The area under the curves.

In systems where large variations in the local ventilation efficiency exist, the slope should not be used as a measure of the efficiency. The area under the curves is the best measure of the efficiency, because all information about the ventilation process is taken into account. The area is a 'smoothed' measure that evens out fluctuations in the data obtained. The ratio between concentrations can be used to estimate the efficiency based on the area under the curves.

The staionary phase

Two approaches are possible:

The direct method; in which the actual conditions under which the ventilation system is to operate are simulated experimentally. The stationary efficiency is then measured directly.

The indirect method; where the stationary efficiency is calculated from parameters obtained from transient methods.

With the prerequisite that the pollution source is effectively passive and is spatial homogenous, then the ventilation efficiency at steady state can be estimated from a transient analysis. If the pollution source is non homogenous then the ventilation efficiency at steady state can be estimated from a transient analysis, if the spatial distribution of the initial concentration corresponds to the distribution of the pollution source. This is possible to arrange theoretically but in practice is very difficult.

There is probably no shortening of the measuring period when the indirect method is applied instead of the direct method.

Acknowledgements

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Nomenclature

A = The age distribution defined in (5)

a_{11}, a_{21} = The components in the eigenvector to λ_1

a_{12}, a_{22} = The components in the eigenvector to λ_2

C = Generic symbol for concentration

$\bar{C}$ = The mean concentration in the room

C_o, $C(0)$ = The initial concentration

C^s = The steady state concentration

C^{tr} = The transient concentration

D = The diffusivity

K_1, K_2 = Constants

$\dot{m}$ = The source of pollution

m = The amount of tracer gas injected

n = The nominal air exchange rate defined in (4)

n_j = The local air exchange rate defined in (8)

Q = The ventilation air flow

r = The local ventilation rate defined in (7)

$\underline{u}$ = The velocity vector (u, v, w)

V = The volume of each zone

V_r = The room's total volume

$\underline{x}$ = The coordinate vector (x, y, z)

Greek symbols

α = Proportionality factor

β = Coupling factor

γ = Mixing factor

ΔT = Overtemperature

ε^a = The absolute ventilation efficiency at steady state defined in (3)

Greek symbols

ε^{r} = The relative ventilation efficiency at steady state defined in (1)

$\bar{\varepsilon}^{r}$ = The overall relative ventilation efficiency at steady state defined in (2)

$\hat{\varepsilon}^{r}$ = The transient relative ventilation efficiency defined in (28)

$\hat{\hat{\varepsilon}}^{r}$ = The transient relative ventilation efficiency defined in (34)

$\hat{\bar{\varepsilon}}^{r}$ = The transient overall relative ventilation efficienc defined in (29)

θ = The local residence-time defined in (6)

λ_1, λ_2 = Eigenvalues

σ_1, σ_2 = Dimensionless factors in the eigenvalues λ_1 and λ_2

τ = Time

τ_o = Specified time interval

∇ = The nabla operator

Suffixes

j = Arbitrary point in the room

f = Exhaust air

t = Supply air

0 = The location of the pollution source

$\left.\begin{array}{l}0.3\\0.9\\1.5\\2.1\end{array}\right\}$ = Measuring levels

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FIGURES

FIGURE 1. Smoke visualization of a ventilation system in operation.

FIGURE 2. Example of the concentration decay in the exhaust air terminal, C_f, and at an arbitrary point, C_j. The mean concentration $\bar{C}$ is also indicated.

FIGURE 3. The sign of the absolute ventilation efficiency ε_j^a for different 'directions' of charge of concentration with regard to the initial concentration C(0).

FIGURE 4. Flow diagrams for the model ventilation system.

FIGURE 5a. Transient behaviour of the 'displacement' system for complete mixing ($\beta = \infty$) and for $\beta = 0$ and $\beta = 0.2$.

FIGURE 5b. Transient behaviour of the 'short circuiting system' for complete mixing ($\beta = \infty$) and for $\beta = 0.2$ and $\beta = 2.0$.

FIGURE 6. A comparison between the transient relative ventilation efficiency, $\hat{\varepsilon}_j^r$, and the relative ventilation efficiency at steady state, ε_j^r, for different locations of the pollution source, $\dot{m}$ ($^{\circ}$ = the upper zone, $_{\circ}$ = the lower zone, $^{\circ}_{\circ}$ = the whole room) and different type of ventilation systems.

FIGURE 7. Measured values by method 2 of the transient relative efficiency $\hat{\varepsilon}_j^r$ against the nominal air-exchange rate $n\Delta T$ is the overtemperature in the supply air relative the mean temperature in the occupied zone.

TABLES

TABLE 1. The local ventilation rates r_f, r_j and the local air-exchange rates n_f, n_j in the control volumes, each with volume v. Systems a, b and c as defined in text.

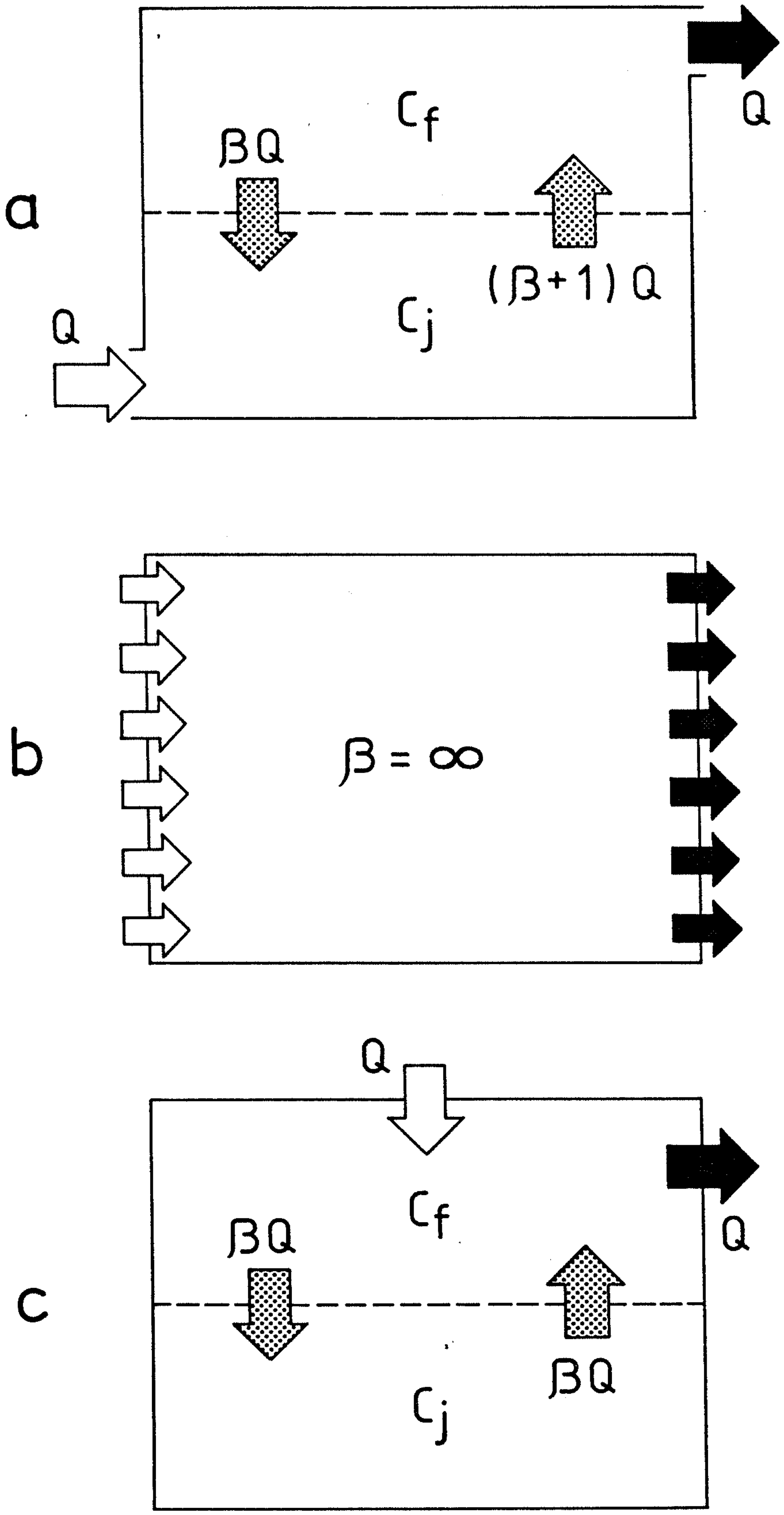
TABLE 2. The eigenvalues $n\sigma_1$ and $n\sigma_2$ for $\beta > 0$.

TABLE 3. The stationary pollution concentration C_j^s in the zone of occupance for different locations of the pollution source $\dot{m}$. V_r is the total room volume.

TABLE 4. The stationary relative ventilation efficiency ε_j^r for different locations of the pollution source.

TABLE 5. The transient relative ventilation efficiency $\hat{\varepsilon}_j^r$.

TABLE 6. Measured values of the relative ventilation efficiency $\hat{\varepsilon}_j^r$ at different levels, j. The room height is 2.7 m.


a
Q
C_f
βQ
C_j
(β+1) Q
Q
b
β = ∞
c
Q
C_f
βQ
Q
C_j
βQ

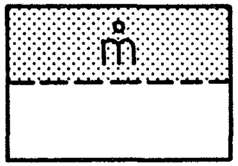
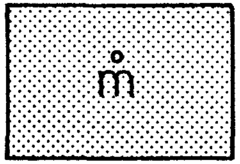
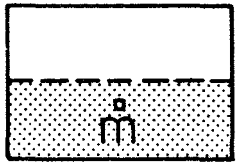
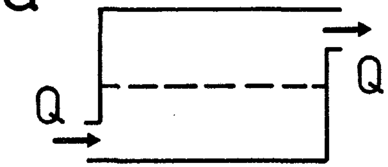
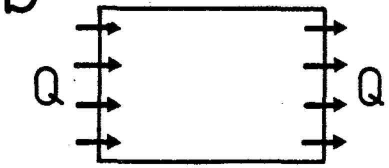
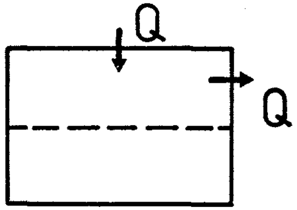
SYSTEM	LOCATION OF THE POLLUTION SOURCE		
			
a 	$\frac{\beta}{\beta+1}\;\frac{\mathring{m}\cdot V_r}{Q}$	$\frac{2\beta+1}{2\beta+2}\;\frac{\mathring{m}\cdot V_r}{Q}$	$\frac{\mathring{m}\cdot V_r}{Q}$
b 	$\frac{\mathring{m}\cdot V_r}{Q}$	$\frac{\mathring{m}\cdot V_r}{Q}$	$\frac{\mathring{m}\cdot V_r}{Q}$
c 	$\frac{\mathring{m}\cdot V_r}{Q}$	$\frac{2\beta+1}{2\beta}\;\frac{\mathring{m}\cdot V_r}{Q}$	$\frac{\beta+1}{\beta}\;\frac{\mathring{m}\cdot V_r}{Q}$

Table 3

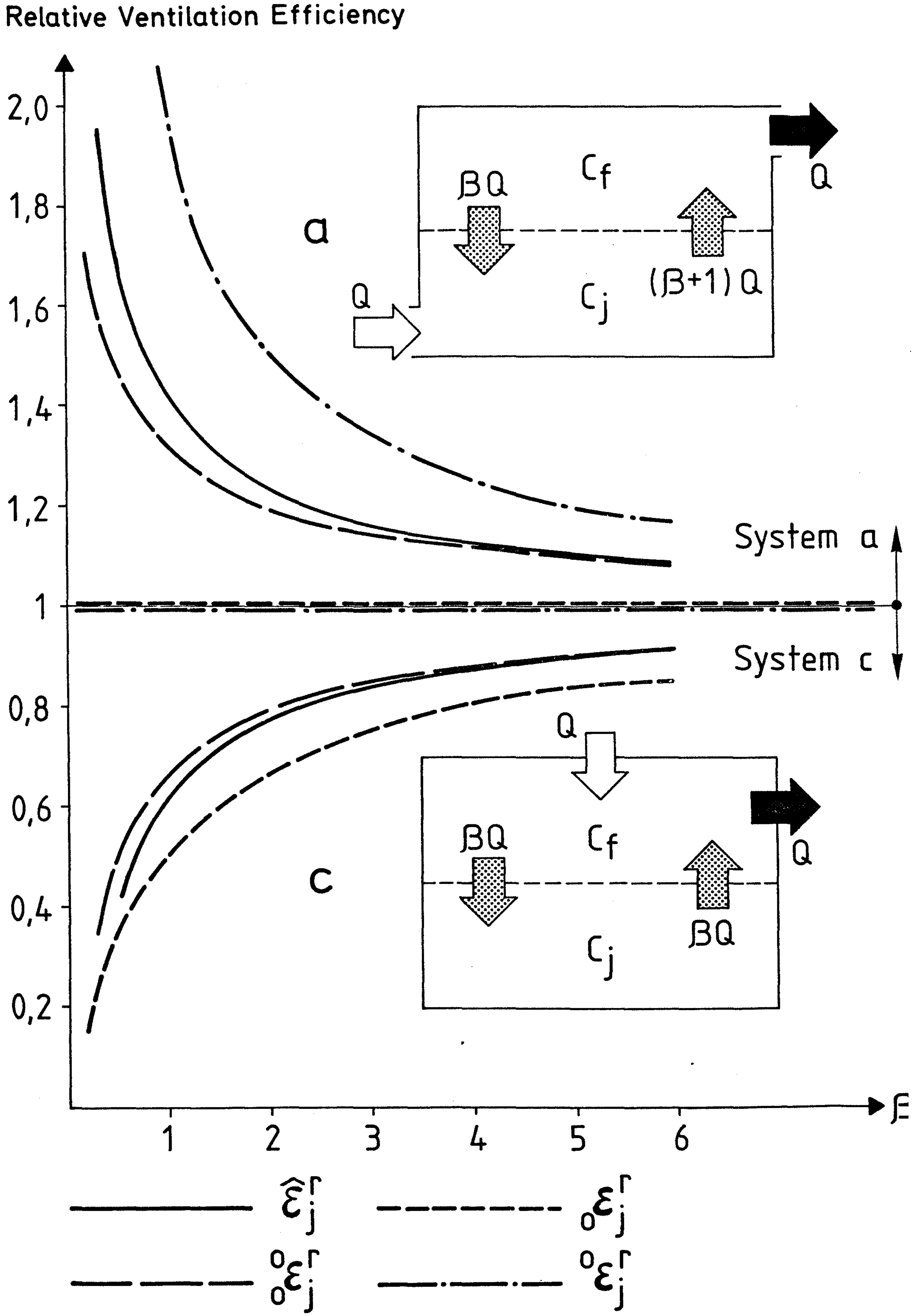

Relative Ventilation Efficiency
2,0
1,8
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
1
2
3
4
5
6
β
a
c
System a
System c
Q
βQ
C_f
C_j
(β+1)Q
Q
Q
βQ
C_f
C_j
βQ
Q
$\hat{\varepsilon}_j^\Gamma$
${}_0\varepsilon_j^\Gamma$
${}_0^0\varepsilon_j^\Gamma$
${}^0\varepsilon_j^\Gamma$

Fig. 6

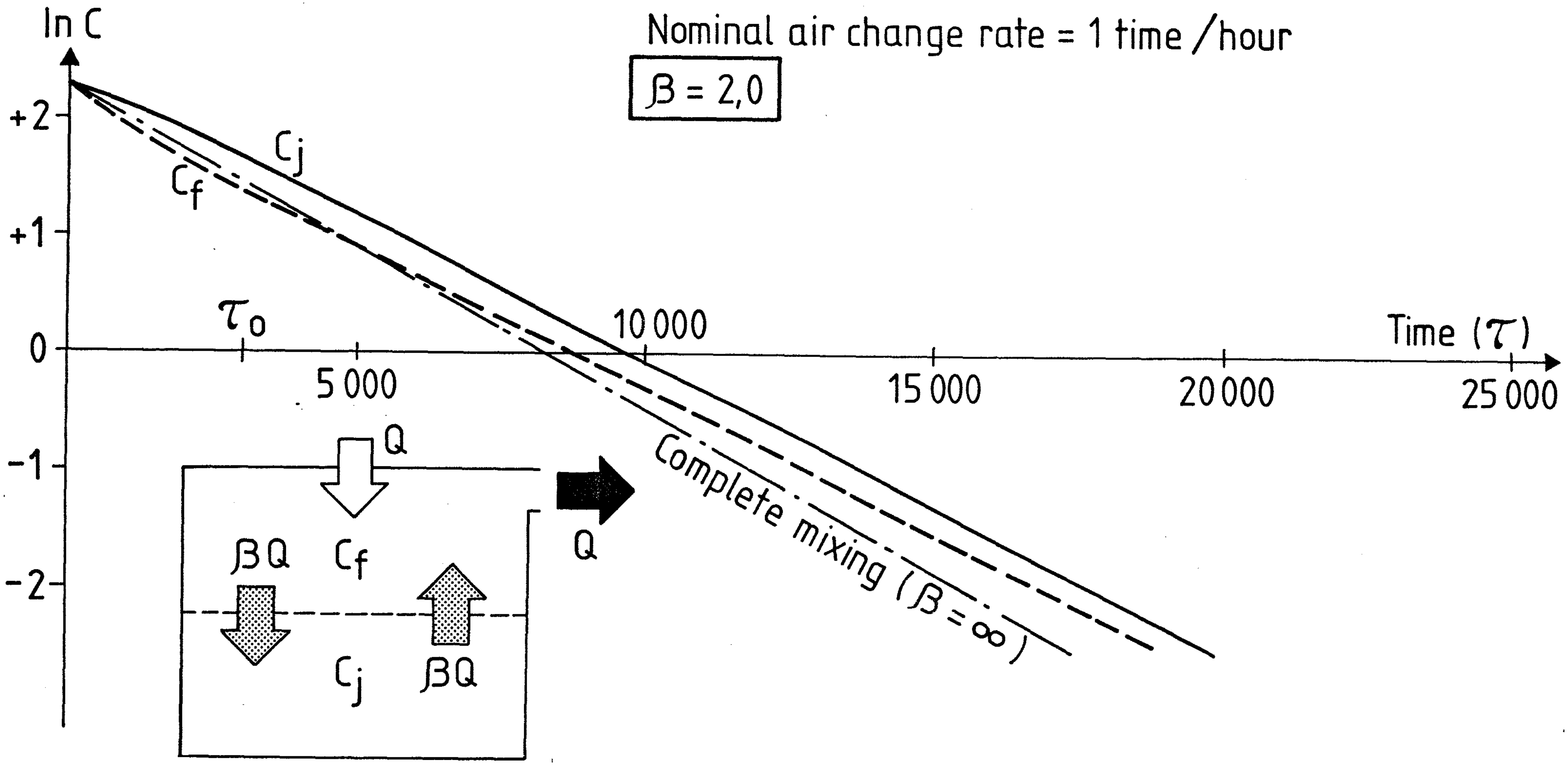
Nominal air change rate = 1 time /hour
β = 2,0
ln C
+2
+1
0
-1
-2
C_j
C_f
τ_o
5 000
10 000
15 000
20 000
25 000
Time (τ)
Complete mixing (β = ∞)
Q
βQ
C_f
C_j
βQ
Q

Figure 56 (2)

[illegible]

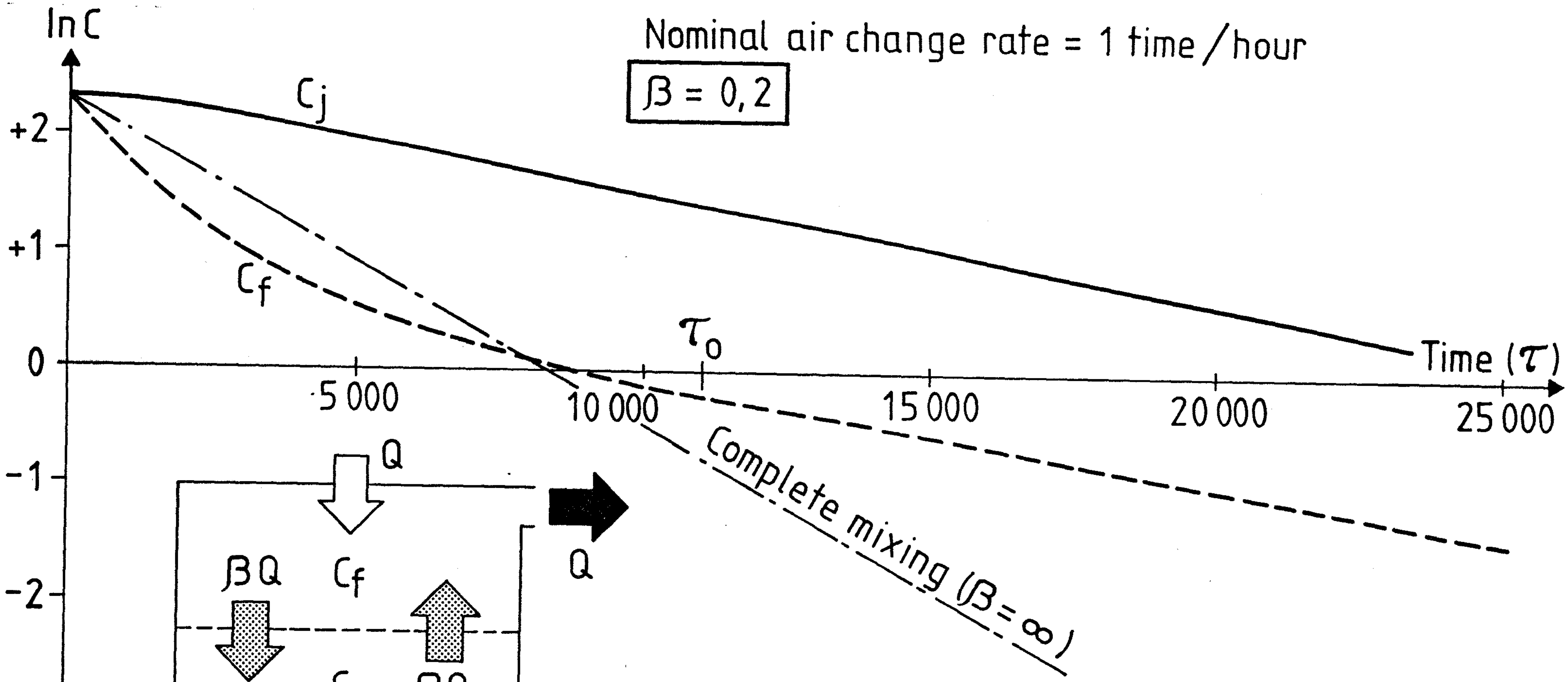
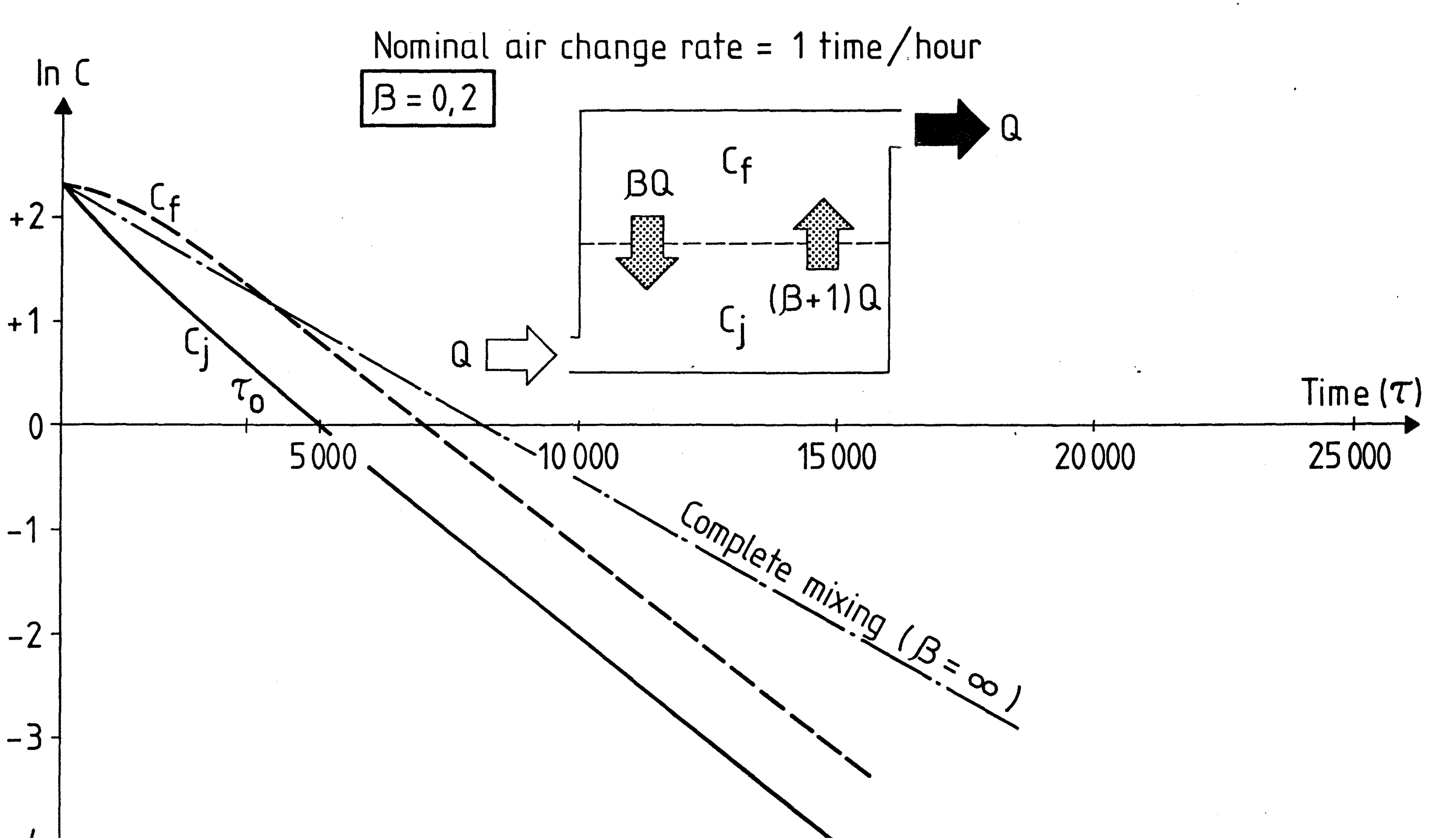
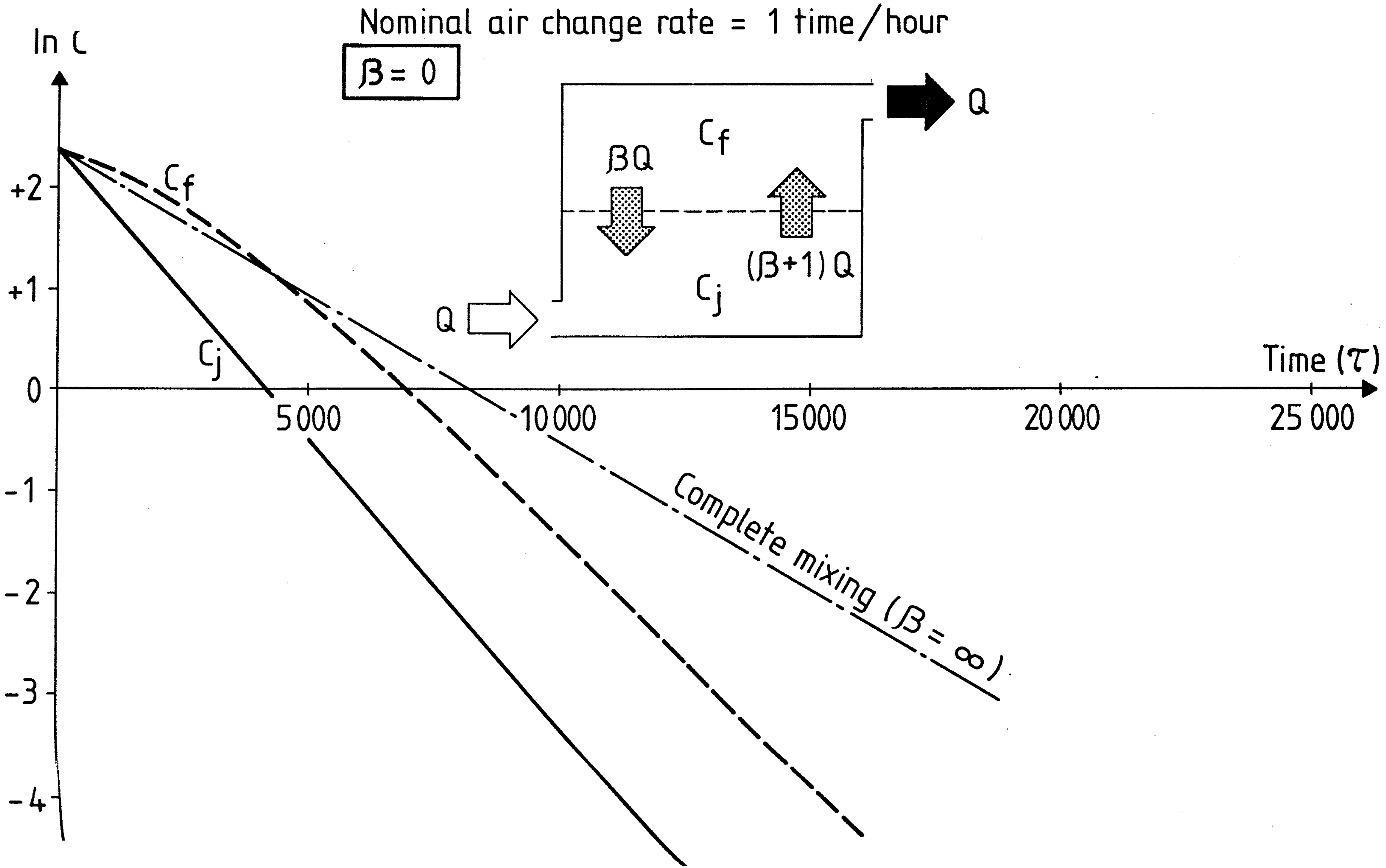
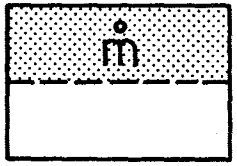
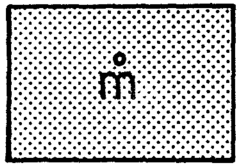
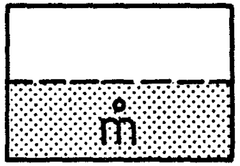
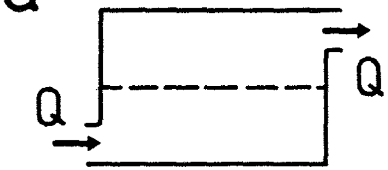
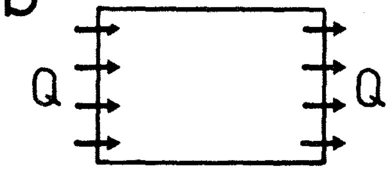
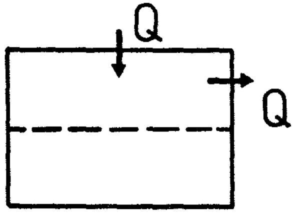
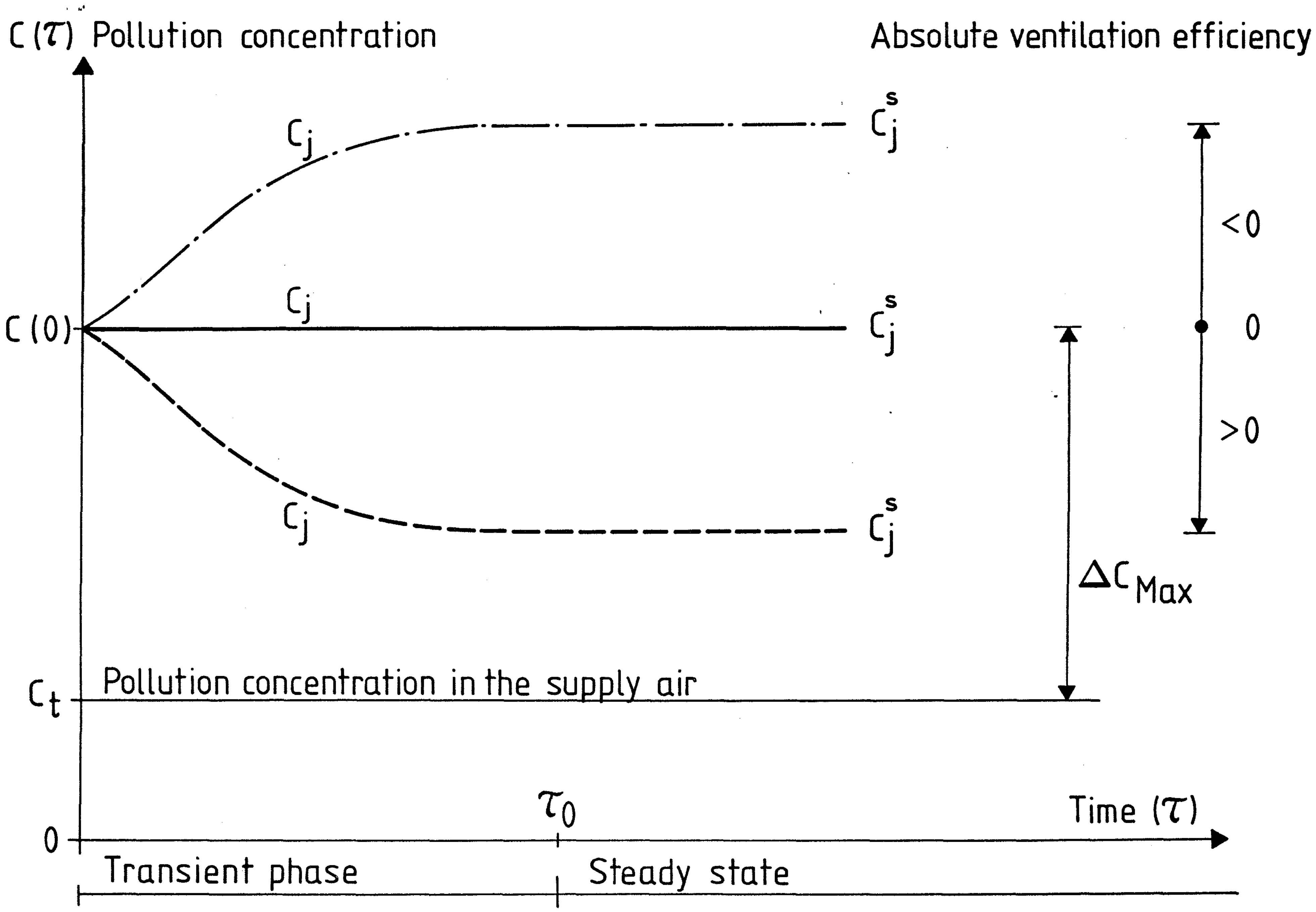

ln C
+2
+1
0
-1
-2
Nominal air change rate = 1 time / hour
β = 0, 2
C_j
C_f
τ_o
Time (τ)
5 000
10 000
15 000
20 000
25 000
Complete mixing (β = ∞)
Q
Q
βQ
βQ
C_f
C_j

Fig 55


Nominal air change rate = 1 time/hour
$\beta = 0,2$
ln C
+2
+1
0
-1
-2
-3
Time (τ)
5000
10 000
15 000
20000
25 000
C_f
C_j
τ_o
Complete mixing ($\beta = \infty$)
Q
βQ
C_f
C_j
$(\beta+1)Q$
Q


Nominal air change rate = 1 time/hour
β = 0
ln C
+2
+1
0
-1
-2
-3
-4
Time (τ)
5000
10000
15000
20000
25000
C_f
C_j
Complete mixing (β = ∞)
Q
βQ
C_f
(β+1) Q
C_j
Q

SYSTEM	LOCATION OF THE POLLUTION SOURCE		
			
a 	$\frac{(\beta+1)}{\beta}$	$\frac{2(\beta+1)}{2\beta+1}$	1
b 	1	1	1
c 	1	$\frac{2\beta}{(2\beta+1)}$	$\frac{\beta}{\beta+1}$


C (τ) Pollution concentration
Absolute ventilation efficiency
C (0)
C_j
C_j
C_j
C_j^s
C_j^s
C_j^s
<0
0
>0
ΔC_Max
C_t
Pollution concentration in the supply air
0
τ_0
Time (τ)
Transient phase
Steady state

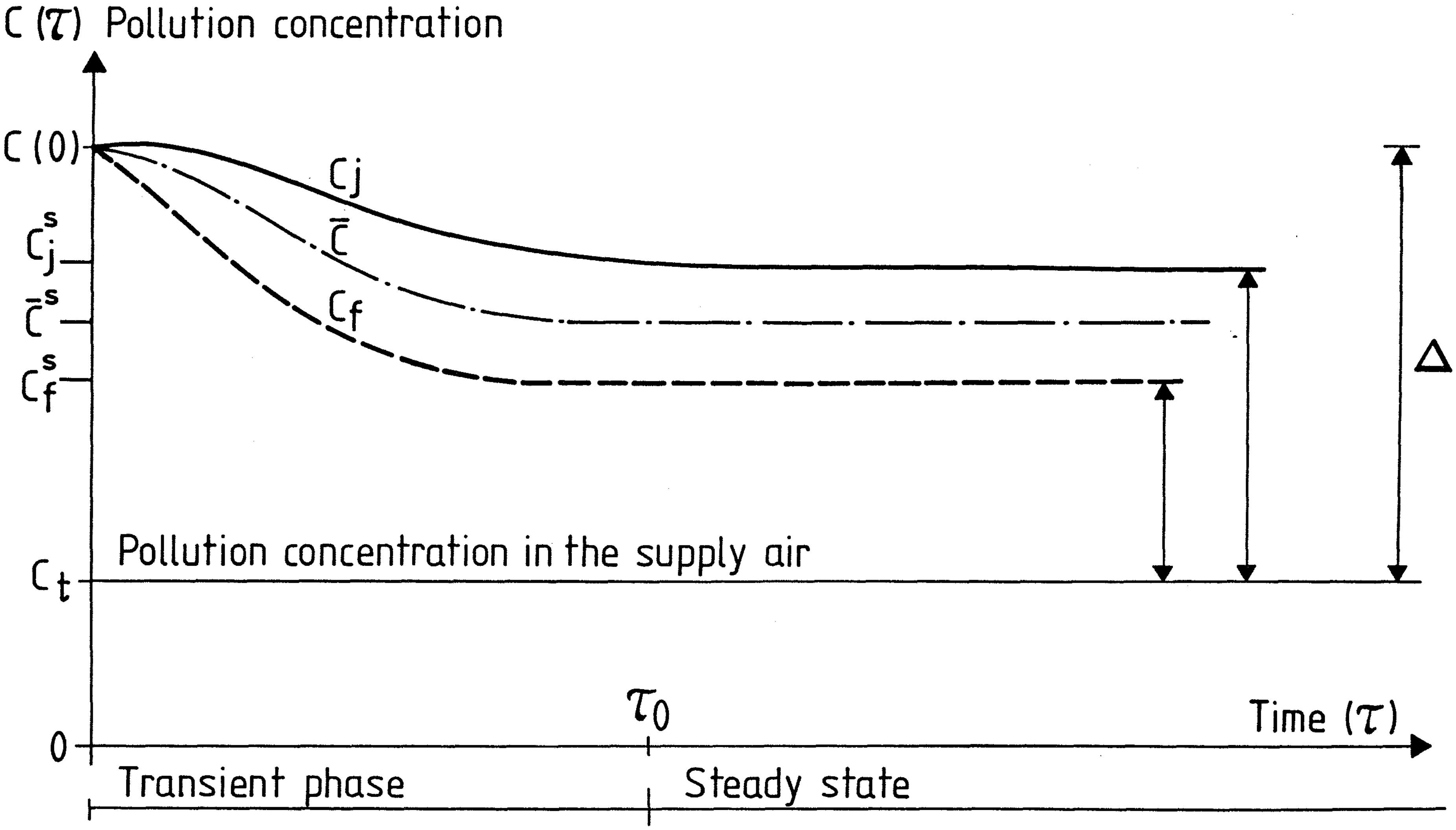

C (τ) Pollution concentration
C (0)
C_j^s
$\bar{C}^s$
C_f^s
C_t
0
C_j
$\bar{C}$
C_f
ΔC Max
Pollution concentration in the supply air
τ 0
Time (τ)
Transient phase
Steady state

Figure 2.