APPLICATION OF MATHEMATICAL MODEL FOR THE BUILDUP OF CARBON MONOXIDE FROM CIGARETTE SMOKING IN ROOMS AND HOUSES

This study uses Turk's equation to obtain curves of concentration vs. time in a room in an office building, and in a one-family dwelling. Results were compared with other similar studies and with current Threshold Limit Values and ambient air quality standards for carbon monoxide. It was found that the model was apparently valid for carbon monoxide and probably valid for other gaseous contaminants not affected by absorption or deposition. Some conclusions were drawn as to minimum ventilation rates for maintaining safe CO levels in the situations studied.

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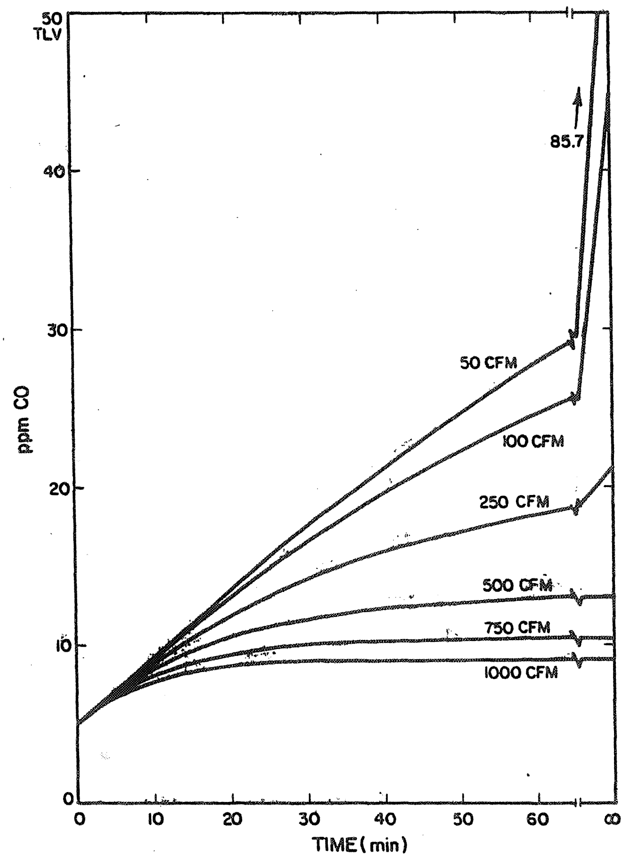

Fig. 1. Situation I, Case 1. 3000 ft³ Meeting Room with 25 people, all smokers, and 100 cigts./hr. (mixing Factor 1/3)

Recently considerable attention has been given to carbon monoxide (CO) as an atmospheric pollutant. Special attention has been called to the contribution of cigarette smoking to the concentration of CO in confined spaces.[1]

Cigarette smoke is not the only source of CO in this situation. All human beings produce and give off CO as a normal part of body metabolism. Although this amount may be small, it is sufficiently large to be taken into consideration in making calculations of the buildup of CO in confined spaces.

Several studies of both a theoretical and an analytical nature have been conducted.[2, 4, 5, 6] Three of these have utilized Turk's Equation, or a modification, to obtain the level of CO or other materials at a given point in time.[4, 5, 6] Because the results obtained using the equation agreed quite well with measured concentrations, it seemed logical to apply the equation to give curves of concentration *vs.* time. This paper presents several curves obtained using Turk's Equation as a mathematical model in representative situations.

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These curves apply primarily to the exposure of a nonsmoker within a given confined space. The exposure of a smoker is quite different and a model to include the smoker would have to include various measurements or estimates of physiological variables which are outside the scope of this paper. No inferences with regard to smoker exposure are or should be drawn from the results of this study. Further, no inferences or conclusions should be drawn regarding the medical significance of exposures to either smokers or nonsmokers.

CALCULATION OF CO CONCENTRATIONS

Calculations were based on Turk's Equation as used by Owens and Rossano.[4] The equation and its variables are:

$$C = (Co\exp-((Qi + E\,Qr)mt/v)) + \left(\frac{CiQi+G}{Qi+EQr}\right)(1-\exp-((Qi + EQr)mt/v))$$

Where v = volume of room in cu. ft.
t = time in minutes
C = concentration in mg/cu. ft. of vapor in room at any time
Co = initial concentration in mg/cu. ft. of vapor in room
Ci = concentration in mg/cu. ft. of vapor in ventilation air
E = efficiency of filter if present (0 to 1.00)
Qi = volume rate of ventilation, cfm
Qr = volume rate of air through filter, cfm

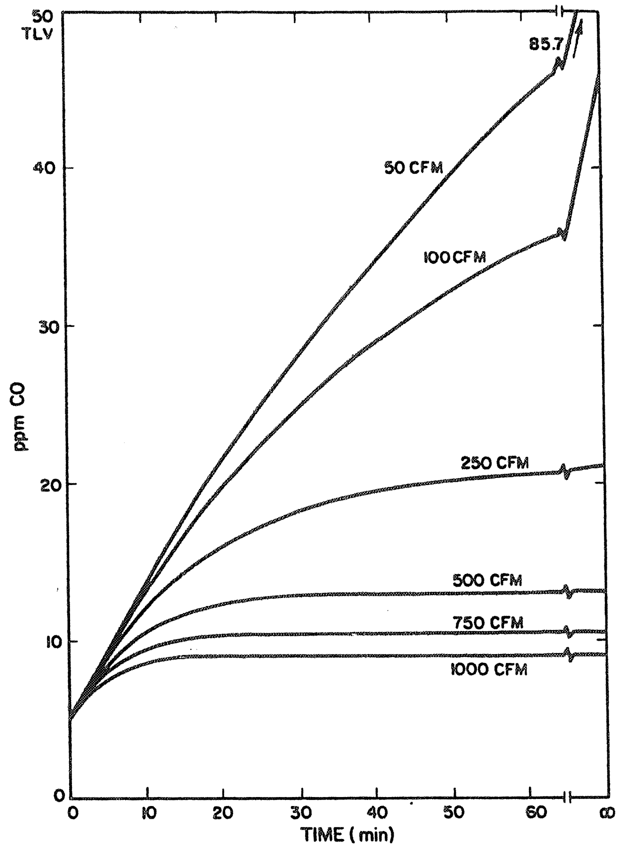

Fig. 2. Situation 1, Case 2. 3000 ft³ Meeting Room with 25 people, all smokers, and 100 cigts./hr. (mixing Factor 2/3)

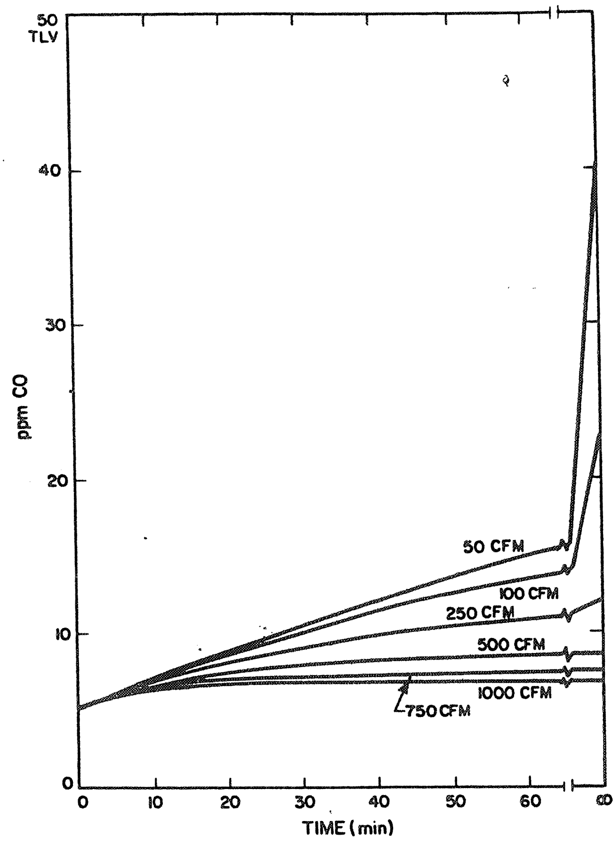

Fig. 3. Situation 1, Case 3. 3000 ft³ Meeting Room with 25 people, 10 smokers, and 40 cigts./hr. (Mixing Factor 1/3)

G = quantity rate in mg/min. of CO generation from cigarettes and body emissions within the room

m = mixing factor within the room

Bridge and Corn[5] suggest that experimental determination of mixing factors will improve agreement between measured and calculated values. Hoegg[6] prefers a modification of the equation to replace Qi with mQi in the denominator of the first expression to the right of the plus sign. Except for his own data Hoegg offers no reasons to make this modification. The data of Bridge and Corn indicate that the equation as given above is suitable. Consequently it was used as written.

The equilibrium concentration at infinite time was calculated from a simplification of the above equation. At t = infinity, the exponential expressions become zero. The equation then becomes:

$$C = 0 + \frac{CiQi + G}{Qi + EQr} \times 1 = \frac{CiQi + G}{Qi + EQr}$$

A further simplification can be made when considering only CO. Since CO is not removed by filtration in most ordinary systems, the EQr terms will be zero and can be deleted.

Table 1 gives the emission rates used in the calculations. The data for body emissions was obtained from Owens and Rossano[4] and the cigarette rates were from the 1964 report to the Surgeon General.[7]

Table 1—Carbon Monoxide Emission Rates

Human Body	17.38 mg/hr./person
Cigarettes	74.0 mg/cigt.

For ease of comparison with other data, all values are reported in ppm. Table 2 shows several conversion factors.

Table 2—Conversion Factors

(25 C, 1 ATM)
Carbon Monoxide

ppm =	31.0 x mg/cu. ft.
mg/cu. ft. =	0.032 x ppm
mg/cu. ft. =	0.028 x mg/cu. m.
mg/cu. m. =	35.3 x mg/cu. ft.

Two basic situations were used. First a room in an office building or other area that might be used for meetings; and second, a home as described in Lefcoe and Inculet.[3] The conditions in each situation were varied within reasonable limits to give sets of data representative of possible conditions which might be ordinarily met. An ambient CO concentration of 5.1 ppm was assumed for both situations (approx. = Los Angeles 1965 average).

For the purposes of this study several obvious sources of CO other than ventilating air, body emissions and smoking were ignored. Some examples might be heaters, cooking and various manufacturing or laboratory processes. In any detailed analysis and modeling of a specific situation these factors would have to be taken into account.

Situation I

In this situation a 3000 cu. ft. room was assumed. It was further assumed that it was a meeting room and that 25 people were going to occupy it. The following variables were used:

number of people—25
number of smokers—25, 10, 0
ventilation (cfm)—50-1000
cigts./pers./hr.—4
mixing factor—1/3, 2/3
time (min.)—0-60

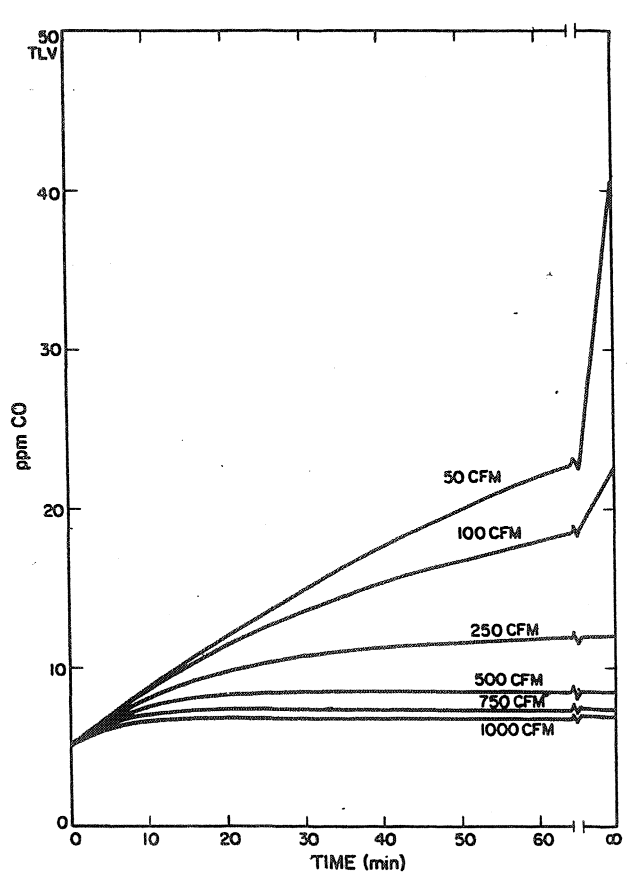

Fig. 4. Situation I, Case 4. 3000 ft³ Meeting Room with 25 people, 10 smokers, and 40 cigts./hr. (Mixing Factor 2/3)

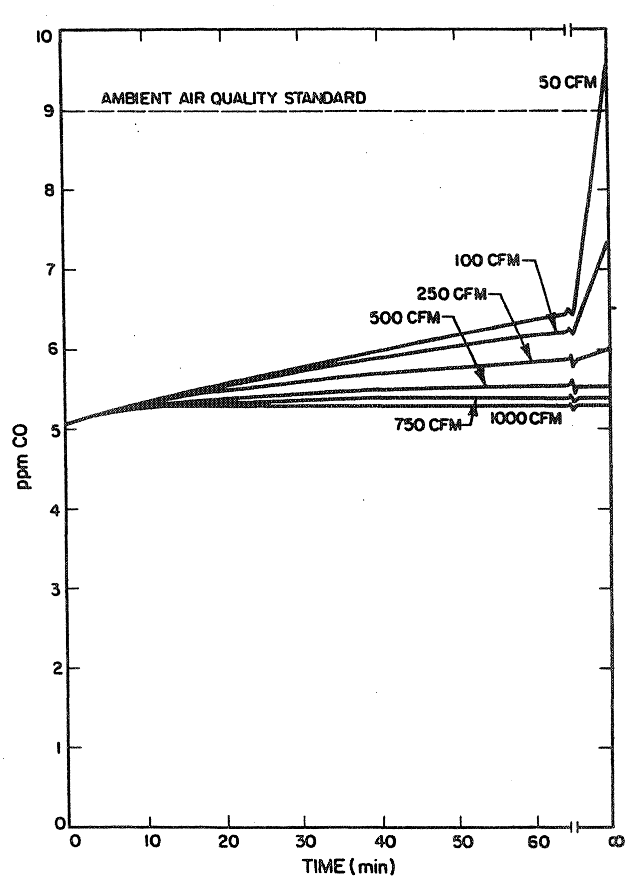

Fig. 5. Situation I, Case 5. 3000 ft³ Meeting Room with 25 people, no smokers, and no smoking (Mixing Factor 1/3)

These variables were combined to give five study cases.

Fig. 1 shows case 1 for situation I for CO buildup.

Case 2, shown in Fig. 2 with a mixing factor of 2/3, showed a more rapid approach to the equilibrium values. The current Threshold Limiting Value (TLV) set by the American Conference of Governmental Industrial Hygienists for CO is 50 ppm.(8) Ambient air quality standards (AAQS) as set forth by the Environmental Protection Agency are even lower, 9 ppm.(9) It is obvious that with any air circulation of 250 cfm of more, the final equilibrium concentration will be less than the TLV. Circulation of about 50 cfm would represent "normal" leakage through doors and/or windows.(6)

Cases 3 and 4 represent more "usual" conditions with 10 of the 25 persons smoking at any one time. Fig. 3 shows CO concentration for Case 3. It is obvious that only if there were less than 50 cfm air flow could the concentration of CO exceed the TLV. However, as with the 25-smoker case, in Case 4 only the time to reach equilibrium differs (*i.e.*, more rapid) from Case 3. This is shown in Fig. 4

Case 5 represents no smokers. Fig. 5 shows the buildup of CO. For all practical purposes, at ventilation rates over 100 cfm no buildup would be observed. In fact, at any rate of ventilation, it would probably require several hours to get a measurable buildup. In all of the cases of Situation I, the initial concentration was taken as an average urban environmental value.

Situation II

In this situation, a house of 20,000 cu. ft. was considered, the same situation used by Lefcoe and Inculet.(3) Their house had a ventilating system rated and measured at 1000 cfm, with an electrostatic filter rated at 90% efficiency at 1000 cfm. For the purposes of these calculations the house was studied over a 21-hour period including the time of a party. The following variables were used in the equation:

number of people—4 usually; 14 for party
number of smokers—2 usually; 7 for party
volume of house—20,000 cu. ft.
air flow (recirculation)—1000 cfm
air flow (make-up)—50 cfm
number of cigts./hr./pers. = 2

Fig. 6 shows the CO concentration during 20 hours. After the party, one hour was alloted to clean up, etc. It was then assumed that no more smoking occurred and the occupants went to bed. The equilibrium values indicate that even during the party the maximum ambient air quality standard was not exceeded (8.9 ppm). It should be noted that buildup in the main party room may exceed the whole house values calculated above.

DISCUSSION

In the first situation the effect of the mixing factor is obvious from the data. At any time, up to equilibrium, the nominal concentration will increase with increasing mixing factor. However, the equilibrium concentration will not change. It would appear then that time of occupancy must be considered in addition to the other factors normally associated with design criteria. The first case in Situation I represents a nearly maximal condition. It allows only 120 cu. ft. per person and about double the "normal" cigarette consumption by all occupants. Even under these conditions it can be seen that 5 cfm per person outside air is adequate to ensure less than 50 ppm of CO at equilibrium. Recirculation at any rate will not alter this requirement since there is no filtration effect on CO. Looking at Cases 3 and 5, for an intermediate condition, and an opposite extreme, it can be seen that 5 cfm per person will not only maintain a level well below the TLV but in fact will approach the level set

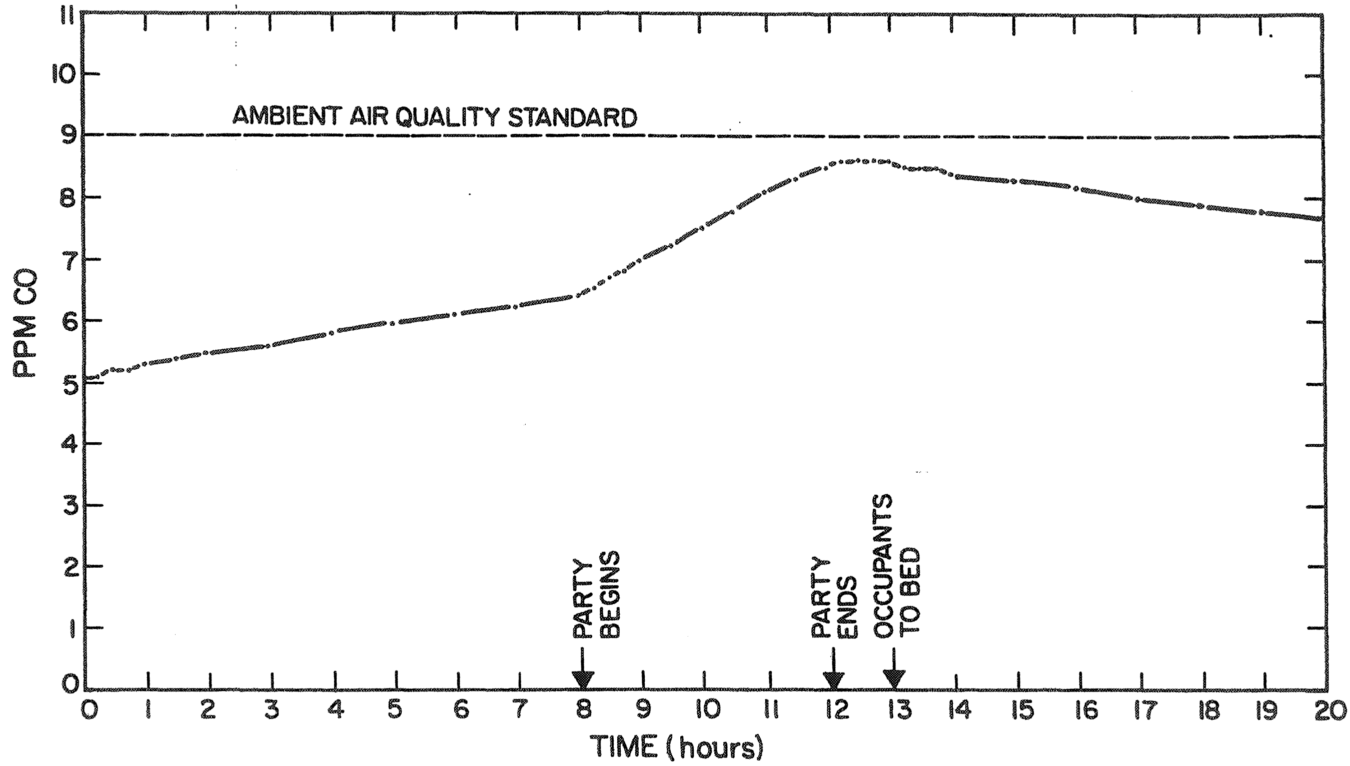

Fig. 6. Situation II. 20,000 ft³ Home, 20-hr. period with varying occupancy and smoking (Mixing Factor 1/3)

for ambient air quality. A factor frequently mentioned in connection with environmental air and ventilation is the number of air changes per hour. As Bridge and Corn[5] point out, this is tied to the mixing factor in a real situation. For Situation I, the flow rates used would give from 1 to 20 turnovers per hour without accounting for mixing factor. To calculate the effective changes one simply multiplies by the mixing factor. For Situation I, the minimum number of effective changes per hour would have to be 1.67, assuming 1/3 as a valid mixing factor. The direct comparability of the calculated and experimental values of Bridge and Corn with the data of this study shows that there is no reason to deny the assumptions used in this study.

The nature of Hoegg's[6] treatment of data makes exact comparison with other data nearly impossible. However, it appears that his use of a sealed chamber model has caused his results to be higher than one would expect in comparison with this study and other studies.

In Situation II, the house shows some interesting features. There the bulk of the air will be recirculated under all conditions. Even after a 4-hour party with 14 people, the maximum level of CO did not quite reach the ambient air quality standard. It is noteworthy that the decay of CO concentration is quite slow. Assuming that the party ended at midnight, by 8 a.m. the next morning there would only be a 1 ppm loss. However, the first 8 hours of the study indicate that about 7 ppm will be the equilibrium concentration under normal conditions. Going to the extreme, it was possible to evaluate the conditions required to exceed the TLV of 50 ppm in 1 hour. It was found that 1000 cigarettes would be required to do that. This would be by any combination of people from 1 person @ 1000 cigarettes/hr. to 1000 people @ 1 cigt./hr. There is no reasonable combination which might exist to produce a 50 ppm level in one hour.

The value of 50 cfm outside air appears to be on the low side for the party. Practically it would probably be higher because of doors being opened and closed at various intervals and the possible opening of windows.

The most logical reference points for a house would be the ambient air quality standards. A case could be stated

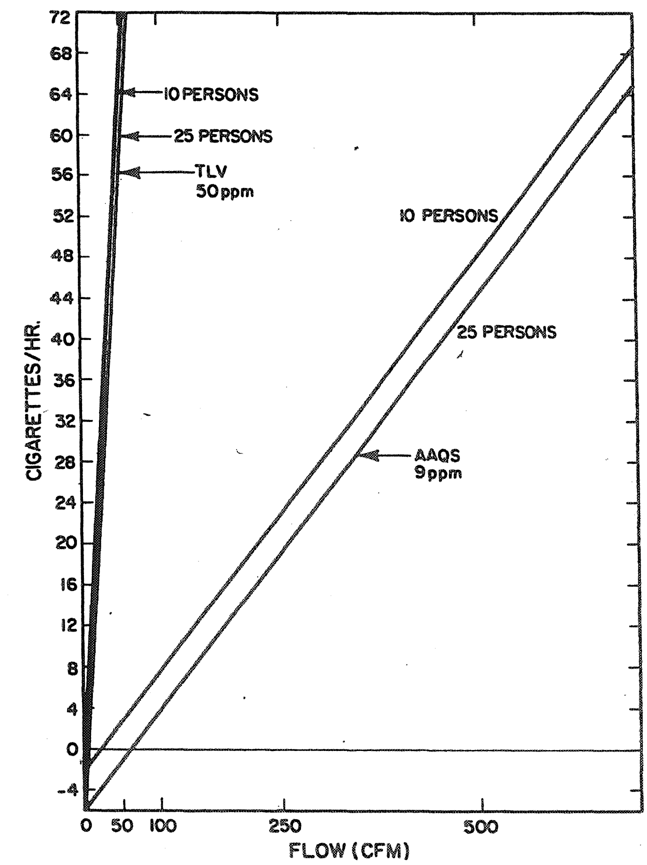

Fig. 7. Maximum allowable cigarettes consumption as a function of flow rate (Qi)

Table 3—Suggested Minimum Ventilations

Location	Occupancy	Smokers	Total Cigts./Hr.	Fresh Air CFM/ Person	Maximum Allowable Conc. (Criteria)
(Volume					
Meeting Room (3000 ft³)	25	25	100	10	50 ppm (TLV)
Meeting Room (3000 ft³)	25	10	40	5	50 ppm (TVL)
Home (2000 ft³)	4	2	4	10	9 ppm (AAQS)
Home (2000 ft³)	>4	>2	>4	15	9 ppm (AAQS)

for the use of TLVs during special events such as parties. However, the slow decay of non-filterable materials such as CO and the continuing occupancy would appear to obviate most such arguments.

A rearrangement of the equation for equilibrium conditions will permit the calculation of the maximum cigarette consumption rate allowable for various flow rates (Qi) when a maximum value for CO is stipulated.

Cigts./hr. = FQi — 0.23 p

Where: Qi = Flow rate in CFM

F = 0.81 X (max. allowed CO—input air CO), in mg

p = number of persons

Two important points now become obvious. First, at equilibrium the calculations are independent of the time and the volume of the space considered. Second, since the relation is now linear the effects of the terms in the equation can be easily seen. Fig. 7 shows a plot of the maximum allowable cigarette consumption versus flow rates, for the TLV (50 ppm) and AAQS (9 ppm). Of interest is the calculation that a closed space occupied by 25 persons must have at least 60 CFM ventilation to meet the AAQS at equilibrium. Effective turnovers per hour could be substituted for CFM. In any case, the use of this equation, or a graphic representation, will permit ventilation requirements to be readily evaluated for virtually any situation.

CONCLUSIONS

Carbon monoxide buildup due to cigarette smoking in rooms and houses can be reasonably modeled using Turk's Equation, as cited by Owens and Rossano.[4] It would be feasible to utilize this same model for any component which is not affected by absorption or deposition. Other components of cigarette smoke are less easily dealt with due to physical or chemical properties. As examples one might consider particulate matter and oxides of nitrogen. Particulate matter is influenced by deposition, adsorption and agglomeration, all of which are dependent on the nature of the environment. Items such as furniture, carpeting and draperies must be considered variables together with temperature and the ventilation parameters. In a recirculating system filtration becomes an important factor. In the case of oxides of nitrogen, adsorption, absorption and chemical reaction must be considered. These factors are also largely dependent on the environment. It has been suggested that CO might be used as an indicator of overall contamination from cigarette smoke[5] because it is relatively inert and free from the phenomena mentioned above. Certainly these considerations make it almost ideal for an initial model on a simple system. Whether or not it can in fact be used as an indicator, any model built around CO should serve well as a starting point for modeling more complex situations.

Levels within current federal standards can be maintained in most commercial environments by allowing at least 5 CFM fresh air per occupant, at maximum expected occupancy. This is based on the current TLV of 50 ppm for CO.

A value of 50 CFM fresh air appears to be adequate for a one-family dwelling. However, more rapid clearance could be effected after parties, or other large gatherings, if the ventilation rate were increased.

Table III shows suggested ventilations for the maintenance of TLV and/or ambient air quality standards.

The data from this study coincide with data from other studies in suggesting that CO concentrations in an occupied space, resulting from cigarette smoking, do not present an inhalation hazard to the non-smoker. □□

REFERENCES

1. The Health Consequences of Smoking, A Report of the Surgeon General: U.S. Dept. HEW, PHS, 1972, DHEW Publication No. (HSM) 72-7516.

2. Harke, H. P., On the Problem of "Passive Smoking," *Muench. Med. Wochschr.*, Vol. 122, No. 51, p. 2328 (1970) (in German.)

3. Lefcoe, N. M. and I. I. Inculet, Particles in Domestic Premises, *Arch. Environ. Health, 22*, p. 230 (1971).

4. Owens, D. F. and A. T. Rossano, Design Procedures to Control Cigarette Smoke and Other Air Pollutants, Paper Presented at ASHRAE Semi-Annual Meeting, Chicago, January 27-30, 1969, 10 pp.

5. Bridge, D. P. and Corn. M., Contribution to the Assessment of Exposure of Nonsmokers to Air Pollution from Cigarette and Cigar Smoke in Occupied Spaces, *Environmental Research, 5*, p. 192, (1972).

6. Hoegg, U. R., The Significance of Cigarette Smoke in Confined Spaces, Masters Thesis, University of Cincinnati, (June, 1972).

7. U.S. Public Health Service, Smoking and Health. A Report of the Advisory Committee to the Surgeon General of the Public Health Service: U.S. Dept. HEW, PHS Publication 1103, January, 1964.

8. Threshold Limit Values for Airborne Contaminants. Adopted by ACGIH for 1972. In "American Conference of Governmental Industrial Hygienists." Cincinnati, Ohio.

9. Fed. Reg. *36*, 8187 (30 April, 1971).