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A MATHEMATICAL MODEL FOR PREDICTING ATTIC VENTILATION RATES REQUIRED FOR PREVENTING CONDENSATION ON ROOF SHEATHING

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ABSTRACT

A mathematical model for predicting the heat transfer and moisture-transfer processes in residential attic spaces is presented. This model is utilized to predict attic ventilation rates required for preventing condensation or frost accumulation on the underside of roof sheathing. Attic ventilation charts are developed covering a wide range of outdoor temperatures, ceiling thermal resistances, and ceiling air penetration rates. The effectiveness of a ceiling vapor barrier is investigated. The effect of indoor humidification on the required attic ventilation rate is examined. Using measured data of Hinrichs, attic ventilation rates predicted by the mathematical model are converted into net free ventilation areas for soffit venting. These values are subsequently compared with the attic ventilation requirements of ASHRAE and the HUD Minimum Property Standards.

INTRODUCTION

It is generally accepted by experts on moisture problems in residential buildings that insufficient attic ventilation may lead to a condensation problem in the attic space during the winter. In the living space below, the household activities of the occupants release as much as 11 kg (25 lbs) of water per day [1] for average American homes. A portion of this water diffuses through the ceiling into the attic space. Another portion is carried into the attic space by air that penetrates through cracks where building materials are joined, and through other cracks such as around lighting fixtures, interior partition walls, and an attic scuttle door. Air penetration into an attic from the space below has been shown to be an important mechanism for transferring water vapor into an attic [2,3]. Once water vapor is introduced into an attic, unless it is ventilated to the outdoor environment, it may condense as water or frost on the cold surface of the underside of a roof. A thick frost layer on the underside of roof sheathing is shown in Fig. 1. During warm periods, this ice layer may melt and drip downward, wetting insulation installed in the attic floor. Water may penetrate light fixtures and leave water marks on the ceiling below. The roof sheathing may absorb moisture, making it susceptible to fungus decay. Eventually, it may be necessary to replace rotted portions of a roof. Decayed roof sheathing is shown in Fig. 2.

Experts on attic moisture problems agree that insufficient attic ventilation can lead to the foregoing problems. However, they disagree on the amount of attic ventilation required to prevent such problems. Current guidelines on the minimum amount of attic ventilation necessary for preventing condensation in attic spaces, as given in the ASHRAE Handbook of Fundamentals [4] and the HUD Minimum Property Standards [5], are based on the experience and knowledge of persons in the building industry; they were developed approximately 30 years ago when the ceilings of houses had very limited insulation. Current recommendations on insulation levels [6] require much larger amounts of ceiling insulation. Houses with highly insulated ceilings will have colder attics which may be more susceptible to condensation. The adequacy of current ventilation requirements for attics having highly insulated ceilings has been questioned.

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Older houses were often constructed without a ceiling vapor barrier. Ceiling insulation has been added to many of these houses. A controversial issue is whether a ceiling barrier should be required as part of such a retrofit.

MATHEMATICAL ANALYSIS

Condensation occurs on a surface when its temperature is below the dew-point temperature of air in contact with it. In an attic of a residence, detrimental condensation or frost occurs first on the underside of the roof sheathing. Condensation does occur on nails which penetrate the roof sheathing before it forms on the underside of the roof sheathing, but condensation on nails is of little consequence.

Attic condensation becomes a problem when it accumulates or builds up over the winter season. For this analysis, it is assumed that no net moisture will accumulate over a period of a day for which the daily-average temperature of the underside of the roof is above the daily-average dew-point temperature of the attic air. This assumption is based on the premise that under such a condition, a small amount of night condensation will be either evaporated or sublimated during warmer day periods, thereby precluding an accumulation of moisture on the underside of the roof sheathing.

In this section, analytical expressions for a heat and a moisture balance for an attic are derived. These expressions are subsequently solved for the critical attic ventilation rate required to maintain the surface temperature of the underside of the roof sheathing above the dew-point temperature of the attic air.

Assumptions

For the analysis, the following heat-transfer assumptions are made:

1. The thermal conductance and water-vapor permeance of the building materials are considered to be constant;
2. Air within the attic is perfectly mixed. This implies that the air temperature and water-vapor pressures are uniform;
3. Radiation exchange occurs only between the underside of the roof and the attic floor. Radiation between these surfaces and other surfaces such as the attic end walls is neglected;
4. Heat and moisture transfers are treated as steady-state processes; and
5. Diffusion transfer of moisture through cracks and openings is negligible compared with convection transfer.

The Heat Balance

Using the attic as a control volume, a heat balance is performed on node 1 of the attic schematic shown in Fig. 3. The rates of heat gain into the attic space by way of convection from the attic floor (ceiling) and air penetration into the attic from the space below are set equal to the rates of heat loss by convection to the roof, heat conduction through the soffit region and attic end walls, and heat loss due to the exchange of attic with outdoor air, or:

$$A_c \cdot h_f \cdot (T_f - T_a) + \dot{V} \cdot A_c \cdot C_p \cdot \rho \cdot (T_i - T_a) = A_r \cdot h_r \cdot (T_a - T_r) + A_s \cdot (T_a - T_o)/R_s + A_e \cdot (T_a - T_o)/R_e + I_a \cdot \rho \cdot C_p \cdot V_a \cdot (T_a - T_o) \quad (1)$$

where

T_i, T_a, T_f, T_r, T_o = daily-average temperatures of the indoor air, attic air, surface of attic floor, underside of the roof, and outdoor air, respectively, °C (°F)

A_c, A_r, A_s, and A_e = surface areas of the ceiling, roof, soffit region (eaves), and attic end walls, respectively, m^2 (ft^2)

I_a = attic ventilation rate, volume changes per hour

ρ = density of air, kg/m^3 (lb/ft^3)

C_p = specific heat of air, $J/kg \cdot K$ ($Btu/lb \cdot °F$)

V_a = volume of the attic space, m^3 (ft^3)

$\dot{V}$ = rate of air penetration into the attic from the space below per unit ceiling area, m^3/h per m^2 (ft^3/h per ft^2), and

h_f, h_r = convection heat-transfer coefficients for the attic floor and underside of the roof, respectively, $W/m^2 \cdot K$ ($Btu/h \cdot ft^2 \cdot °F$).

Performing a heat balance on a unit area of attic floor (node 2 of the attic schematic shown in Fig. 3), the rate of heat conduction through the ceiling is equal to the rates of convective heat loss to the attic air and the net radiation exchange between the attic floor and the underside of the roof, or

$$(T_i - T_f)/R_c = h_f \cdot (T_f - T_a) + F \cdot (T_f - T_r) \tag{2}$$

where

R_c = thermal resistance of the attic floor (ceiling), $m^2 \cdot K/W$ ($h \cdot ft^2 \cdot °F/Btu$), and

F = radiation heat-transfer coefficient between the attic floor and the underside of the roof, $W/m^2 \cdot K$ ($Btu/h \cdot ft^2 \cdot °F$).

The other symbols are as previously defined. Here the radiation exchanges between either the underside of the roof or the attic floor and other surfaces in the attic such as attic end walls have been neglected. The thermal resistance of the ceiling (R_c) does not include the air film resistance at the attic floor.

The radiation heat-transfer coefficient (F) is defined by the relation:

$$F = E \cdot \sigma \cdot (T_f^2 + T_r^2) \cdot (T_f + T_r) \tag{3}$$

where

E = emittance factor (see equation 13), and

σ = Stefan-Boltzmann constant, $W/m^2 \cdot K^4$ ($Btu/h \cdot ft^2 \cdot °R^4$).

Performing a similar heat balance on a unit area of the underside of the roof (node 3 of the attic schematic shown in Fig. 3), the rate of heat conduction through the roof is equal to the rates of convective heat gain from the attic air and the net radiation exchange between the attic floor and the underside of the roof, or

$$(T_r - T_o')/R_r = h_r \cdot (T_a - T_r) + F \cdot (T_f - T_r) \tag{4}$$

where

T_o' = daily-average sol-air temperature of the exterior surface of the roof, °C (°F) and

R_r = thermal resistance of the roof, $m^2 \cdot K/W$ ($h \cdot ft^2 \cdot °F/Btu$).

The thermal resistance of the roof (R_r) does not include the air film resistance at the underside of the roof. The radiation heat-transfer coefficient (F) is defined by Eq. 3.

The daily-average sol-air temperature (T_o') of the exterior surface of the roof is defined below:

$$T_o' = T_o + \alpha \cdot H/h_o - (1 - C_o) \cdot \varepsilon \cdot Q/h_o \tag{5}$$

where

α = solar absorptance of the exterior roof surface

H = mean daily-average solar radiation incident on the roof, $W/m^2(Btu/h \cdot ft^2)$;

h_o = convection heat-transfer coefficient, $W/m^2 \cdot K(Btu/h \cdot ft^2 \cdot °F)$;

ε = emittance of exterior surface of the roof;

Q = net radiation exchange between a black-body surface and the sky, $W/m^2(Btu/h \cdot ft^2)$; and

C_o = mean cloud-cover fraction.

The sol-air temperature is that temperature of the outdoor air which, in the absence of all radiation exchanges, gives the same rate of heat entry into the surface as would exist with the actual combination of incident solar radiation, radiant energy exchange with the sky and other outdoor surroundings, and convective heat exchange with the outdoor air. It should be pointed out that Eq. 5 does not take into account differences in incident solar radiation due to variations in roof slope.

Eqs. 1, 2 and 4 are three analytic expressions for attic air temperature (T_a), attic floor temperature (T_f), and underside roof temperature (T_r). These three equations may be rearranged as follows:

$$-\frac{A_r}{A_c} \cdot h_r \cdot T_r - h_f \cdot T_f + (h_f + \dot{V} \cdot C_p \cdot \rho + \frac{A_r}{A_c} \cdot h_r + \frac{A_s}{A_c}\frac{1}{R_s} + I_a \cdot \rho \cdot C_p \frac{V_a}{A_c} + \frac{A_e}{A_c}\frac{1}{R_e}) \cdot T_a =$$

$$\dot{V} \cdot C_p \cdot \rho \cdot T_i + (\frac{A_s}{A_c} \cdot \frac{1}{R_s} + I_a \cdot \rho \cdot C_p \cdot \frac{V_a}{A_c} + \frac{A_e}{A_c} \cdot \frac{1}{R_e}) \cdot T_o \quad (6)$$

$$-F \cdot T_r + (1/R_c + h_f + F) \cdot T_f - h_f \cdot T_a = T_i/R_c \quad (7)$$

$$(1/R_r + h_r + F) \cdot T_r - F \cdot T_f - h_r \cdot T_a = T_o'/R_r \quad (8)$$

Eq. 6 has been rewritten in terms of ratios of attic geometry parameters with the idea that these ratios would vary much less than the geometry parameters themselves from one attic to the next. For particular parameter values, these three equations can be solved simultaneously to determine the underside roof temperature (T_r).

<u>The Moisture Balance</u>

Performing a steady-state moisture balance on an attic, the rates of water-vapor transfer by diffusion and air penetration are equal to the loss of moisture by exchange of attic air with outdoor air and the loss of moisture due to diffusion through the roof, the attic end walls, and the soffit region, giving the relation:

$$A_c \cdot M_c \cdot (P_i - P_a) + A_c \cdot \rho \cdot \dot{V} \cdot \omega_i = I_a \cdot \rho \cdot V_a \cdot (\omega_a - \omega_o) + (P_a - P_o) \sum_{i=1}^{3} A_i M_i \quad (9)$$

where

M_c = water-vapor permeance of ceiling, $kg/Pa \cdot s \cdot m^2$ $(lb/h \cdot ft^2 \cdot in\ Hg)$;

P_i, P_a, P_o = water-vapor pressures of the indoor space, attic, and outdoor environment, respectively, Pa(in Hg);

ω_i, ω_o = humidity ratios of indoor and outdoor environments, respectively, kg water/kg dry air (lb water/lb dry air); and

$A_i M_i$ = product of surface area and water-vapor permeance for the attic end wall, soffit region, and roof, $kg/s \cdot Pa(lb/h \cdot in\ Hg)$.

The other symbols are as previously defined.

We seek an attic ventilation rate (I_a) which precludes condensation from occurring on the underside of the roof sheathing. This critical attic ventilation rate can be determined by solving Eq. 9 for I_a and setting the vapor pressure of the attic air (P_a) equal to the saturation vapor pressure corresponding to the underside roof temperature (T_r), or

$$I_a = \frac{M_c \cdot (P_i - P_a) + 0.622 \cdot \rho \cdot \dot{V} \cdot \left(\frac{P_i}{P_{atm} - P_i}\right) - (P_a - P_o) \sum_{i=1}^{3} A_i M_i}{0.622 \cdot \rho \cdot \frac{V_a}{A_c} \cdot \left(\frac{P_a}{P_{atm} - P_a} - \frac{P_o}{P_{atm} - P_o}\right)} \tag{10}$$

Here the psychrometric relation

$$\omega = 0.622 \left(\frac{P}{P_{atm} - P}\right) \tag{11}$$

has been introduced to eliminate humidity ratios from Eq. 10.

Method of Solution

The underside roof temperature (T_r) as given by a simultaneous solution of Eqs. 6, 7, and 8 is a weak function of the attic ventilation rate (I_a). This is because radiation heat transfer is large in comparison with convection heat transfer. Therefore, we can assume an attic ventilation rate and solve for the underside roof temperature. The saturation water-vapor pressure corresponding to this temperature is determined and set equal to the water-vapor pressure of the attic air (P_a). A new attic ventilation rate is calculated from Eq. 10. Using this attic ventilation rate, the process is repeated until convergence is obtained. Only several iterations are required to achieve convergence. An attic ventilation rate greater than the critical value determined from such an iterative procedure should preclude the accumulation of moisture on the underside of the roof sheathing.

PARAMETERS NEEDED FOR THE ANALYSIS

Description of Attic System

The attic system used for the analysis is given in Fig. 4. For this attic system, the width of the soffit region was 0.30 m, (1.0 ft.), and the roof pitch was 7/16. Based on this attic system, the surface areas of the ceiling (A_c), the soffit (eaves) region (A_s), the attic end walls (A_e), and the roof (A_r) were determined and the following ratios were derived:

$A_r/A_c = 1.16$

$A_s/A_c = 0.0667$

$A_e/A_c = 0.187$, and

$V_a/A_c = 1.14$ m (3.73 ft)

The construction details of various components comprising the attic system were selected as follows: the roof consisted of shingles and roofing paper laid on top of 1.3-cm (1/2-in.) plywood sheathing which was nailed to nominal 5 x 15-cm (2 x 6-in.) rafters placed 0.41 m (16 in.) on center; the soffit region consisted of 1.3-cm (1/2-in.) plywood sheathing; attic end walls were comprised of wood-bevel siding attached to 1.3 cm (1/2-in.) insulation sheathing which was nailed to nominal 5 x 10-cm (2 x 4-in.) studs placed 0.41 m (16 in.) on center, and the ceiling consisted of 1.3 cm (1/2-in.) gypsum board attached to nominal 5 x 15-cm (2 x 6-in.) joists placed 0.41 m (16 in.) on center. Exterior wood surfaces of the attic were assumed to be painted with oil-base paint. The interior surface of the ceiling was assumed to be painted with latex paint. Several thicknesses of ceiling insulation were considered (see Table 1).

Heat-Transfer Parameters

Thermal Resistance Values. The thermal resistances of the various components of the attic system were calculated using the series resistance method as outlined in Ref. 4. For these calculations, wood structural members were treated as parallel heat-flow paths. Heat-transfer coefficients for the air films were taken from Ref. 4. Thermal resistance values for the various components of the attic system are summarized in Table 1.

Ceiling Air Penetration Rates. During the winter season, the air inside a residence is warmer and therefore lighter than cooler outdoor air. This causes a buoyant force (stack effect) to be exerted on this air. As a result, the air inside a residence will tend to rise and leak into the attic space through cracks in the ceiling construction, interior partition walls via baseboard cracks, and penetrations at electrical fixtures and an attic scuttle door.

In deriving ceiling air penetration rates for the analysis, it was assumed that the rate of ceiling air penetration was equal to a specified fraction of the overall house exfiltration rate. Ceiling air penetration rates used for the analysis are given in Table 2. These values are based on a house air exfiltration rate of 0.75 volume changes per hour.

For the analysis presented in section 4, the ceiling air penetration rate ($\dot{V}$) was considered to be independent of the outdoor temperature (T_o). It should be pointed out that the stack effect of any given house will increase approximately in proportion to the temperature difference between the indoor and outdoor environments. Therefore, the ceiling air penetration rate will become larger as the outdoor temperature is reduced. This latter effect was not included in the present analysis.

Attic Heat-Transfer Parameters. The convection heat-transfer coefficients (h) in $W/m^2 \cdot K$ at the underside of the roof and at the attic floor were assumed to be governed by the relation:

$$h = 1.5 \cdot (\Delta T)^{0.33} \tag{12}$$

Here ΔT is the surface-to-air temperature difference in degrees Kelvin. This relation is applicable to natural convection heat-transfer in the turbulent regime for heat flow in an upward direction. In attics, air motion due to ventilation is small. The effect of such air motion on the convection heat-transfer coefficient (h) was neglected.

In calculating the radiation heat-transfer coefficient (F), it was assumed that the emittances of the underside of the roof and the attic floor were 0.9. The roof and attic floor were treated as infinite parallel plates, and the emittance factor (E) was calculated using the relation:

$$E = \frac{1}{1/\varepsilon_f + 1/\varepsilon_r - 1} , \tag{13}$$

where ε_f and ε_r are the emittances of the attic floor and the underside of the roof, respectively. The emittance factor (E) was calculated to be 0.82.

Water-Vapor-Transfer Characteristics

Water-vapor-transfer characteristics for the components of the attic system are given in Table 3. The permeance of the latex painted gypsum board is based on a NBS dry-cup measurement performed in accordance with ASTM Standard Method of Test E-96-72 [7]. The roofing shingles were assumed to be very permeable because their lapped configuration would permit water vapor to readily pass through them by air convection. The permeance values for the wood members are based on data contained in Ref. 8 and assumed moisture contents.

Permeance values for the roof, attic end wall, soffit region, and the ceiling with and without a vapor barrier were calculated using the series-resistance method outlined in Ref. 4. For these calculations, wood structural members were treated as parallel water-vapor transfer paths. When a vapor barrier was included in the ceiling, it was assumed that it did not cover the joists. Overall permeance values for the various composite surfaces of the attic system are given in Table 4. The low water-vapor transfer characteristics of the roof are due to the high water-vapor resistance of the glue which bonds together the separate wood layers of the plywood. The use of plywood sheathing is nonetheless quite common. The low water-vapor transfer characteristics of the attic end walls and the soffit region are due to the low permeance of the oil-base paint applied to the exterior surface of these components. This represents a worst-case situation which is nonetheless commonly found in practice.

Exterior Surface Parameters

The mean outdoor relative humidity for the months of Dec., Jan., and Feb. for most parts of the U.S. (except for desert regions and coastal regions in Oregon) ranges from 70 to 80% [9]. For the analysis, the outdoor relative humidity was taken to be 75%. Using the psychrometric chart and taking the outdoor relative humidity to be 75%, outdoor humidity ratios were determined for the outdoor temperature conditions -18° to 4.4°C (0 to 40°F) selected for the analysis.

The heat-transfer parameters used to determine the sol-air temperature are summarized in Table 5. Emittance values for roofing shingles could not be found in the literature. The surface emittance for roofing shingles was estimated to vary between 0.85 to 0.95 [10]. For this analysis, the emittance (ε) for the roof shingles was taken to be 0.90. The mean Jan. cloud cover for much of the U.S. ranges between 60 to 80% [9]. For this analysis, the mean cloud cover (C_o) was taken to be 70%. The solar absorptance (α) is an average of solar absorptance values for roof shingles given in Ref. 11. The outside surface heat-transfer coefficient (h_o) is based on data presented in Ref. 4 for a 4.5-m/s (10-mph) wind condition, which is a representative winter wind speed for many parts of the U.S. [9]. The net radiation sky loss (Q) was calculated using the Bliss sky-radiation data [12] and taking the outdoor relative humidity to be 75%. The mean daily-average solar radiation (H) is a representative mean Jan. value for the northern U.S.

Indoor Conditions

The indoor temperature of the living space below the ceiling was taken to be 21°C (70°F). The indoor relative humidity levels and corresponding indoor vapor pressures used for the analysis are given in Table 6. The relative humidity levels without humidification are based on values for an average house given in Fig. 5 of Chap. 20 of the ASHRAE Handbook of Fundamentals [4]. Values for a humidified house are based on the limiting indoor relative humidity for double-pane glass given in Table 1 of Chapter 5 of the ASHRAE Equipment Handbook [13].

DISCUSSION OF RESULTS

Attic Ventilation Charts

The mathematical model described in section 2 was programmed on a digital computer. Using the heat-transfer parameters given in section 3, the mathematical model was used to predict required attic ventilation rates for a non-humidified house. These results are displayed graphically in Fig. 5 for the case of an air tight ceiling [$\dot{V}$ = 0 $m^3/s \cdot m^2$ (0 cfm/ft^2).], in Fig. 6 for the case of 1/4 of the house exfiltration passing into the attic space [$\dot{V}$ = 1.3 x 10^{-4} $m^3/s \cdot m^2$ (0.025 cfm/ft^2)], and in Fig. 7 for the case of 1/2 of the house exfiltration passing into the attic space [$\dot{V}$ = 2.5 x 10^{-4} $m^3/s \cdot m^2$ (0.050 cfm/ft^2)]. In each figure, the left chart is for a house without a ceiling vapor barrier and the right one for a house with a ceiling vapor barrier.

Another corresponding set of attic ventilation charts for a humidified house are given in Figs. 8, 9, and 10. It is interesting to note that the required attic ventilation rates for a humidified house exposed to an outdoor temperature of -18°C (0°F) having an uninsulated ceiling and having a ceiling air penetration rate ($\dot{V}$) equal to 1.3 x 10^{-4} $m^3/s \cdot m^2$ (0.025 cfm/ft^2) are 0.9 volume changes per hour without a ceiling vapor barrier and 0.7 volume changes per hour with a ceiling vapor barrier. These figures are in agreement with corresponding measured values reported in Ref. 3 in which the required attic ventilation rate for the case of an uninsulated ceiling was found to lie between 1.12 and 1.44 volume changes per hour without a ceiling vapor barrier, and between 0.65 and 0.87 volume changes per hour with a ceiling vapor barrier. These measured data were obtained at an outdoor temperature of -21°C (-5.0°F), an indoor temerature of 21°C (70°F), and indoor relative humidity of 40%.

Effect of Particular Parameters

Consider an example house which is non-humidified, has R-19 ceiling insulation without a ceiling vapor barrier, and is located in a climate where the daily-average outdoor temperature (T_o) is -18°C (0°F). For this house, the ceiling air penetration rate ($\dot{V}$) is 1.3 x 10^{-4} $m^3/s \cdot m^2$ (0.025 cfm/ft^2). This figure corresponds to 1/4 of the overall house exfiltration passing into the attic space and could perhaps be considered a representative figure. From Fig. 6a, the required attic ventilation rate for this house is 2.8 volume changes per hour. The effects of various factors (such as adding a ceiling vapor barrier, increasing the ceiling thermal resistance, reducing the outdoor temperature, and humidifying the house) on the required attic ventilation rate are summarized in Table 7.

It is interesting to note that the addition of a ceiling vapor barrier to the example house reduced the required attic ventilation rate from 2.8 to 1.8 volume changes per hour, or a decrease of 36%. Here it is assumed that the use of a vapor barrier has no effect on the rate of air penetration into the attic from the space below. This is perhaps a good assumption for the case of a vapor barrier being included on the backside of the ceiling insulation, leaving many air passageways in the construction unobstructed. On the other hand, a continuous vapor barrier such as lapped polyethylene probably would provide a reduction in the rate of air penetration into the attic from the space below. Air penetration into the attic from the space below is seen to reduce the effectiveness of a ceiling vapor barrier. A ceiling vapor barrier does permit the required attic ventilation rate to be reduced.

The addition of R-19 insulation to existing R-19 ceiling insulation increased the required attic ventilation from 2.8 to 3.6 volume changes per hour, or an increase of 29%. The addition of more ceiling insulation reduces the temperature of the underside of the roof sheathing and makes it more susceptible to condensation, requiring higher attic ventilation rates. On the other hand, removal of the existing R-19 ceiling insulation completely eliminated the need for attic ventilation. For such a situation, heat transfer into the attic space from the space below is sufficient to maintain the temperature of the underside of the roof sheathing above the dew-point temperature of the attic air. When insulation is added to existing uninsulated ceilings of attics where insufficient ventilation is provided, a condensation problem may develop.

Increasing the outdoor temperature from -18°C (0°F) to -1.1°C (30°F) decreased the required attic ventilation rate from 2.8 to 1.6 volume changes per hour, or a decrease of 43%. If the outdoor temperature (T_o) is based on mean Jan. temperature, such a reduction in outdoor temperature is comparable to moving a house from the center of the U. S. to the far northern part [9]. This result indicates that lower attic ventilation rates should be required for warmer climates than for colder climates. It should be pointed out that in the foregoing comparison, the ceiling air penetration rate ($\dot{V}$) was assumed to remain constant, which may not be strictly correct. The ceiling air penetration rate depends upon the pressure difference across the ceiling due to the stack effect. Moving a house into a colder climate will increase the indoor-to-outdoor temperature difference. This will in turn produce an increase in the stack effect and cause an increase in the ceiling air penetration rate. Under such a condition, more moisture will be transported into the attic. This means that even higher attic ventilation rates would be required in colder climates than these predicted values, owing to an increase in the ceiling air penetration rate, which was not included in the present analysis.

The addition of humidification to the example house increased the required attic ventilation rate from 2.8 to 5.0 volume changes per hour, or an increase of 79%. This result indicates that attic ventilation guidelines should require considerably higher attic ventilation rates for humidified houses as opposed to non-humidified houses.

Adequacy of Current Guidelines

In this section, current attic ventilation requirements of ASHRAE [4] and the HUD Minimum Property Standards (MPS) [5] are compared to corresponding predicted values obtained by using the mathematical model and measured attic ventilation rates of Hinrichs [14].

A house with R-19 ceiling insulation and an air penetration rate ($\dot{V}$) equal to 1.3×10^{-4} $m^3/s{\cdot}m^2$ (0.025 cfm/ft^2) was positioned at the top of ASHRAE Condensation Zones[1/] 1, 2, and 3. Using climatic data contained in Ref. 9, mean daily-average Jan. temperatures of -18°C (0°C) for the top of zone 1, -9.4°C (15°F) for the top of zone 2, and -1.1°C (30°F) for the top of zone 3 were obtained. These temperatures were used as the outdoor temperature (T_o) in the mathematical model. From attic ventilation charts 6 and 9, required attic ventilation rates for cases with and without a ceiling vapor barrier were determined for both a non-humidified

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1/ see Fig. 6, Chap. 20, 1977 ASHRAE Handbook of Fundamentals.

and a humidified house. Using Hinrichs' [14] measured attic ventilation rates (i.e., continuous soffit vents on opposite sides of an attic produce 0.135 $m^3/s \cdot per\ m^2$ (26.5 cfm per ft^2) of net free ventilation area for a wind speed of 4.5 m/s (10 mph)), these predicted ventilation rates were converted into net free ventilation areas required for soffit venting. A wind speed of 4.5 m/s (10 mph) was considered to be a representative winter wind speed for many parts of the continental U.S. [9]. These predicted net free ventilation areas for soffit venting are compared to current ventilation requirements of ASHRAE and the HUD MPS in Table 8.

This comparison shows that the predicted net free ventilation area for soffit venting for a non-humidified house in Condensation Zone 1 approximately corresponds to the requirements of the HUD MPS for cases with and without a ceiling vapor barrier. The predicted requirements indicate that the HUD MPS practice of permitting a ceiling vapor barrier to be omitted in Condensation Zone 1 but requiring twice as much net free ventilation area is approximately correct. Perhaps the ASHRAE requirements should be revised to permit such a practice. The predicted requirements also indicate that current attic ventilation requirements should be increased for humidified houses located in Condensation Zone 1. It should be pointed out that the validity of these results depends upon the accuracy of Hinrichs' measured attic ventilation rates and the assumption that 25% of the overall air infiltration rate for a house penetrates into the attic space.

SUMMARY AND CONCLUSIONS

A mathematical model for predicting the required attic ventilation rate for preventing condensation or frost accumulation on the underside of roof sheathing of a residence was presented. This mathematical model was used to generate attic ventilation charts for both humidified and non-humidified houses covering a wide range of outdoor temperatures, ceiling thermal resistances, and air penetration rates.

The mathematical model was used to investigate the effect of certain factors on the required attic ventilation rate for a non-humidified house having R-19 ceiling insulation and for which 1/4 of the overall house exfiltration rate penetrated into the attic. It was found that the addition of a ceiling vapor barrier reduced the required attic ventilation rate 36%. Air penetration into the attic from the space below was observed to reduce the effectiveness of a ceiling vapor barrier. Adding R-19 insulation to existing R-19 ceiling insulation increased the required attic ventilation rate 39%. An attic with an uninsulated ceiling was found to be always free of condensation even when attic ventilation was completely eliminated because heat transferred through the ceiling maintained the temperature of the underside roof sheathing above the dew-point temperature of the attic air. Raising the outdoor temperature from -18°C (0°F) to -1.1°C (30°F) decreased the required attic ventilation rate 43%. And, humidifying a house increased the required attic ventilation rate 79%.

Using measured data of Hinrichs, attic ventilation rates predicted by the mathematical model were converted into net free ventilation areas for soffit venting. These values were subsequently compared with the attic ventilation requirements of ASHRAE and the HUD MPS. This comparison showed that the predicted net free ventilation area for soffit venting for a non-humidified house in Condensation Zone 1 approximately corresponded to the requirements of the HUD MPS for cases with and without a ceiling vapor barrier. The practice of permitting a ceiling vapor barrier to be omitted in Condensation Zone 1 but requiring twice as much net free ventilation area is approximately correct. The predicted requirements also indicated that current attic ventilation requirements should be increased for humidified houses located in Condensation Zone 1. It was pointed out that the validity of this comparison depends upon the accuracy of Hinrichs' measured attic ventilation rates and the assumption that 25% of the overall air infiltration rate for a house penetrates into the attic space.

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TABLE 1. Thermal Resistances for Components of the Attic System

Component	Total Thermal Resistance $m^2 \cdot K/W$	Total Thermal Resistance $h \cdot ft^2 \cdot °F/Btu$
Roof	0.264	1.50
Soffit Region	0.322	1.83
Attic End Wall	0.628	3.57
Ceiling		
• No Insulation	0.209	1.19
• R-11 Insulation	1.99	11.3
• R-19 Insulation	2.96	16.8
• R-30 Insulation	5.05	28.7
• R-38 Insulation	6.51	37.0

TABLE 2. Ceiling Air Penetration Rates

Fraction of House Air Exfiltration %	Ceiling Air Penetration Rate $10^{-3} m^3/s \cdot m^2$	cfm/ft^2
0	0.0	0.0
25	0.13	.025
50	0.25	.050

TABLE 3. Water Vapor Transfer Characteristics of Components of the Attic System

Component	Permeance $10^{-10} kg/s \cdot m^2 \cdot Pa$	grains/$h \cdot ft^2 \cdot$ in Hg	Source
Latex Paint Applied to 1/2-in. Gypsum Board	8.6	15	NBS Dry-Cup Measurement
Asphalt-Impregnated Kraft Paper	0.14	0.25	[4]
Roofing Paper, 15-lb asphalt felt	3.2	5.6	[4]
1/2-in. Plywood Sheathing	0.52	0.90	[4]
1/2-in. Insulating Sheathing	40.	70.	[4]
Oil-Base Paint	0.40	0.70	[4]
Wood Structural Members			
° 2x4 Studs (M. C.* = 15%)	1.0	1.8	Based on Ref. 8
° 2x6 Rafters (M. C. = 15%)	0.69	1.2	Based on Ref. 8
° 2x6 Joists (M. C. = 12%)	0.35	0.60	Based on Ref. 8
Wood Siding (M. C. = 20%)	15.	26.	Based on Ref. 8

*M. C. denotes moisture content.

TABLE 4. Overall Permeance Values for Composite Surfaces of the Attic Systems

Composite Surface	10^{-10} kg/s·m^2·Pa	grains/h·ft^2·in Hg
Roof	0.43	0.74
Attic End Wall	0.37	0.65
Soffit Region	0.22	0.39
Attic Floor (Ceiling)		
° with vapor barrier	0.17	0.29*
° without vapor barrier	7.5	13.

* In deriving this value, it was assumed that the vapor barrier did not cover the joists. If the vapor barrier covered the joists, the permeance of the ceiling would be 19% smaller than the value cited.

TABLE 5. Summary of Heat-Transfer Parameters Used to Determine Sol-Air Temperatures

Roof Emittance (e)	0.9
Mean Cloud Cover Percent (C_o)	70.
Solar Absorptance (α)	0.83
Exterior Surface Heat-Transfer Coefficient (h_o)	28 W/m^2·K (5.0 Btu/h·ft^2·°F)
Net Radiation Loss to Sky (Q)	68.1 W/m^2 (21.6 Btu/h·ft^2)
Daily-Average Solar Radiation (H)	118. W/m^2 (37.5 Btu/h·ft^2)

TABLE 6. Indoor Humidity Conditions

Outdoor Temp.		Without Humidification			With Humidification		
		rh	Vapor Pressure		rh	Vapor Pressure	
°C	°F	%	10^3·Pa	in Hg	%	10^3·Pa	in Hg
-18.	0	20	.502	.148	30	0.753	.222
-12.	10	27	.678	.200	36	0.902	.266
-6.7	20	34	.851	.251	43	1.08	.318
-1.1	30	42	1.05	.310	50	1.25	.370
4.4	40	49	1.23	.362	59	1.48	.436

TABLE 7. Effect of Various Factors on the Required Ventilation Rate (I_a) for an Example House *

Description of House	I_a h^{-1}	change** %
Example House	2.8	-
Example House Plus Ceiling Vapor Barrier	1.8	-36.
Example House Plus R-19 Ceiling Ins.	3.6	+29.
Example House Less Ceiling Ins.	None	-
Moving Example House into Warmer Climate (T_o = 1.1°C (30°F))	1.6	-43.
Humidifying Example House	5.0	+79.

* The example house is a non-humidified house which has R-19 ceiling insulation without a ceiling vapor barrier, has a ceiling air penetration rate ($\dot{V}$) equal to 1.3 x 10^{-4} $m^3/s{\cdot}m^2$ (0.025 cfm/ft^2), and is located in a climate where the daily-average outdoor temperature (T_o) is -18°C (0°F).

** With respect to example house.

TABLE 8. Required Net Unobstructed Ventilation Opening in m^2 (ft^2) for Soffit Venting for an Example House with a 1200 ft^2 Ceiling Insulated with R-19 Insulation

	Predicted*			
	Non-Humidified House	Humidified House	HUD MPS	ASHRAE**
Condensation Zone 1				
° With a Ceiling Vapor Barrier	0.46 (5.0)	0.76 (8.2)	0.37 (4)	0.37 (4)
° Without a Ceiling Vapor Barrier	0.76 (8.2)	1.30 (14.0)	0.74 (8)	Not Permitted
Condensation Zone 2				
° With a Ceiling Vapor Barrier	0.40 (4.2)	0.52 (5.6)	0.37 (4)	0.37 (4)
° Without a Ceiling Vapor Barrier	0.56 (6.0)	0.78 (8.4)	0.74 (8)	Not Permitted
Condensation Zone 3				
° With a Ceiling Vapor Barrier	0.34 (3.6)	0.40 (4.2)	0.37 (4)	0.37 (4)
° Without a Ceiling Vapor Barrier	0.42 (4.6)	0.56 (6.0)	0.74 (8)	0.37 (4)

* Using the Mathematical Model in conjunction with Hinrichs' measured attic ventilation rates.

** From Table 3, Chap. 20, 1977 ASHRAE Handbook of Fundamentals.

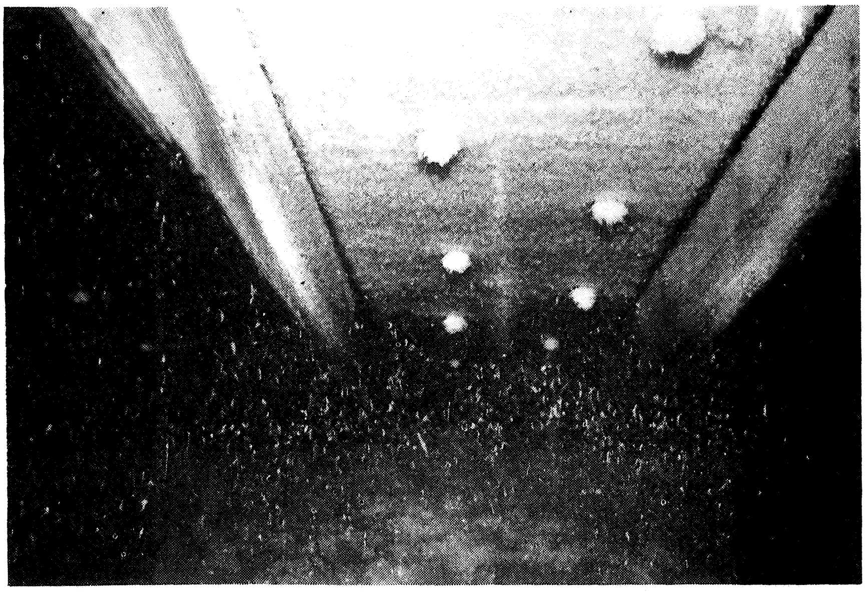


Fig. 1 Illustration of frost build-up on the underside of roof sheathing. (This photograph was provided by Vince Meyer of Diversified Insulation.)

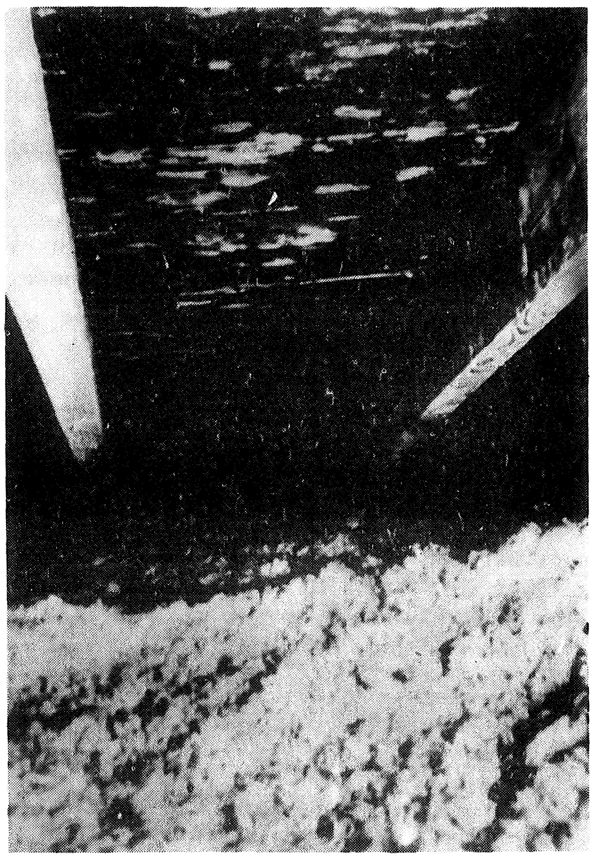


Fig. 2 Illustration of fungus rot on the underside of roof sheathing. (This photograph was provided by Vince Meyer of Diversified Insulation.)

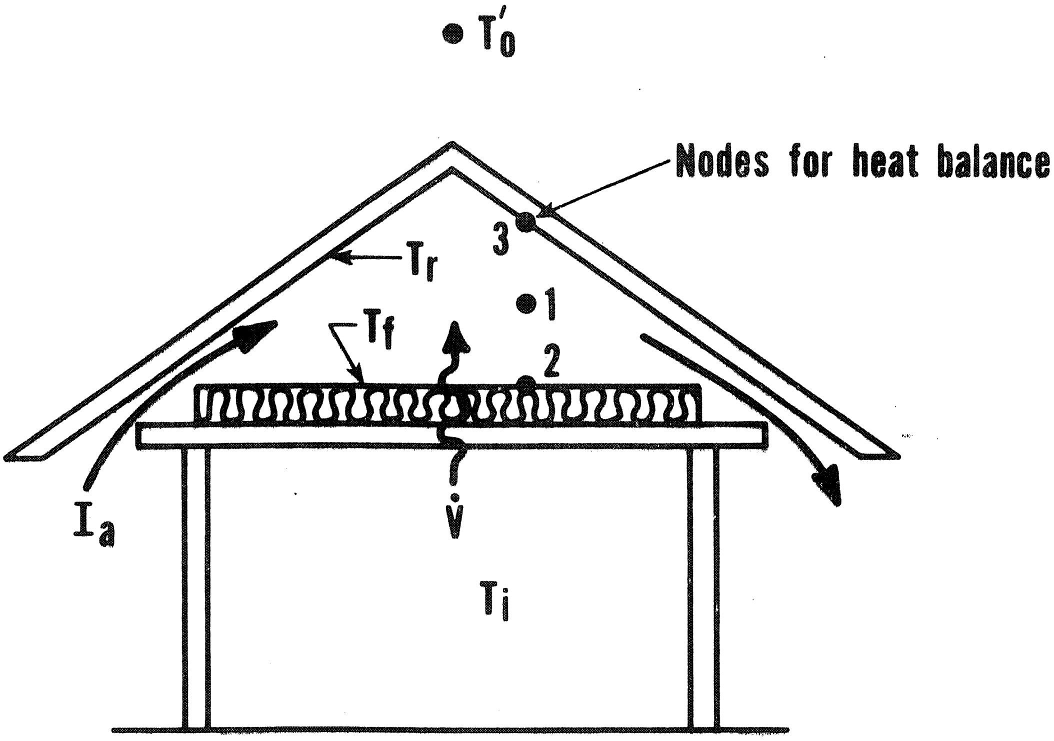


Fig. 3 Schematic of attic

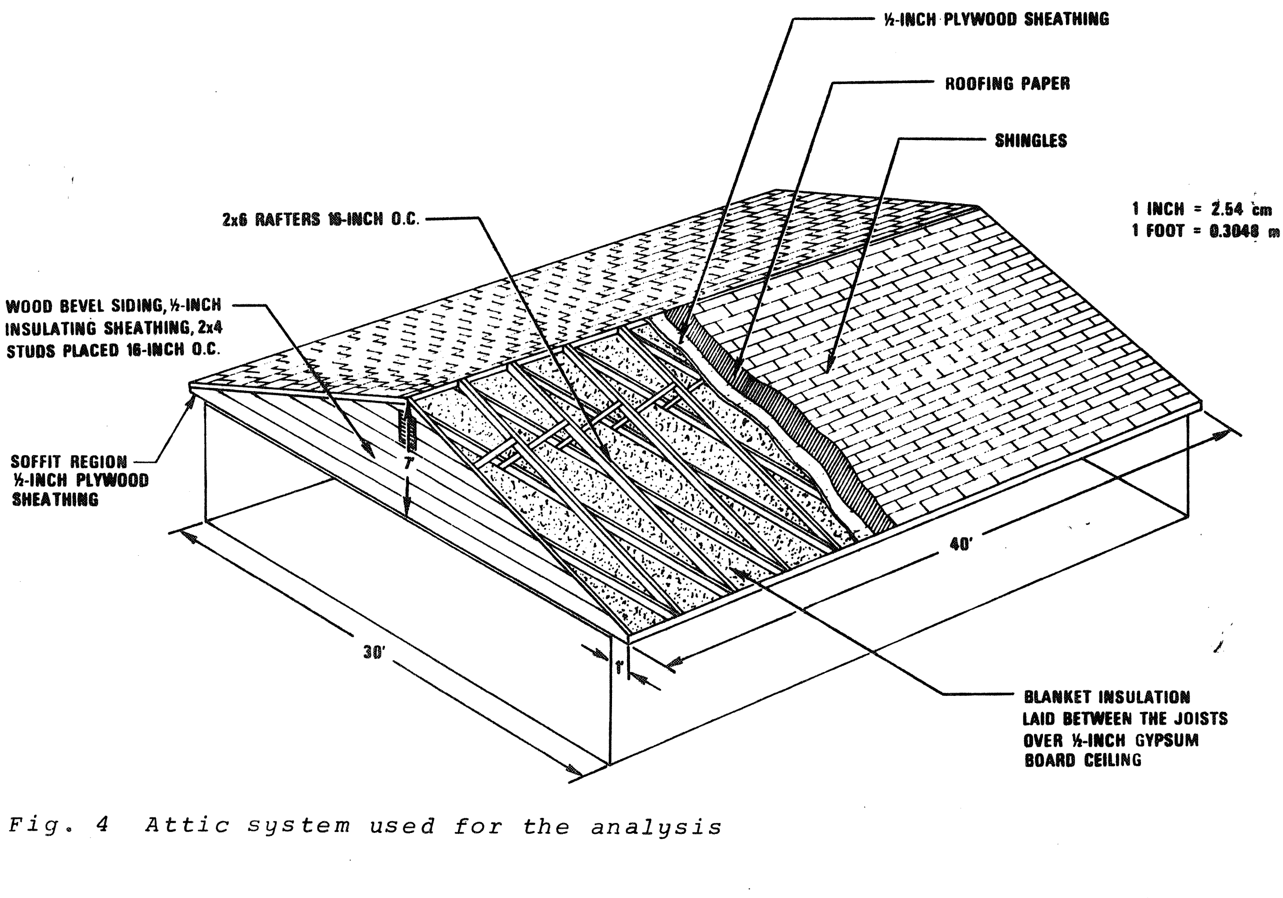
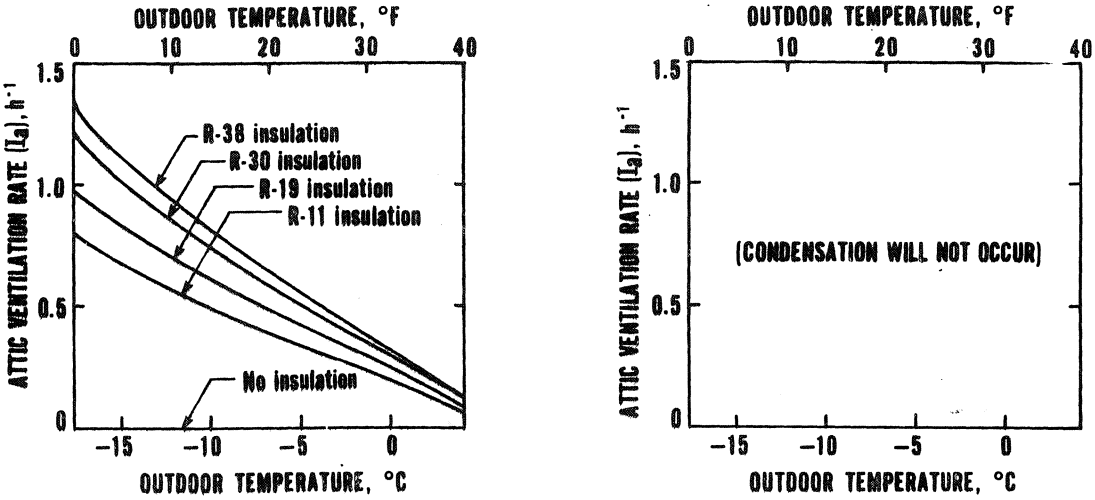
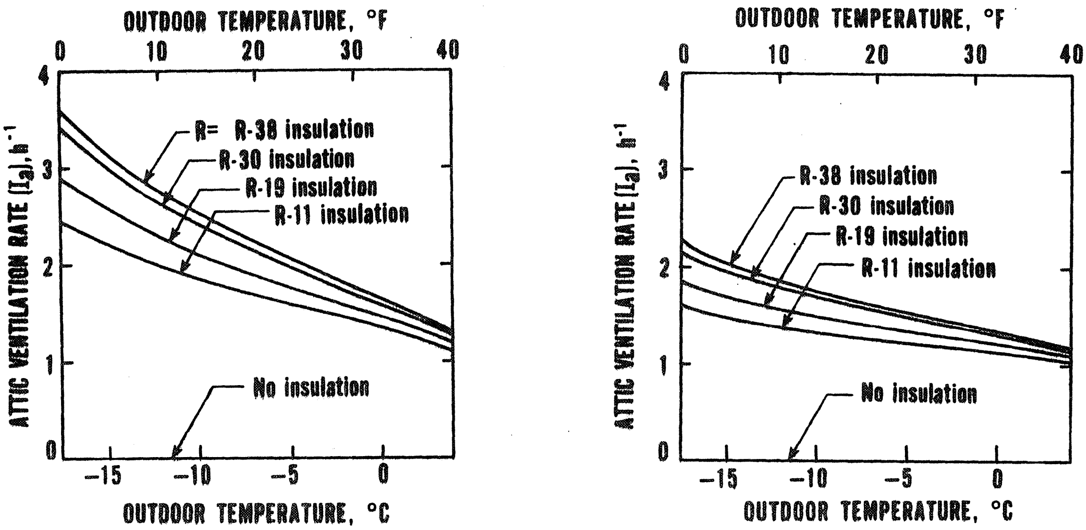


Fig. 4 Attic system used for the analysis



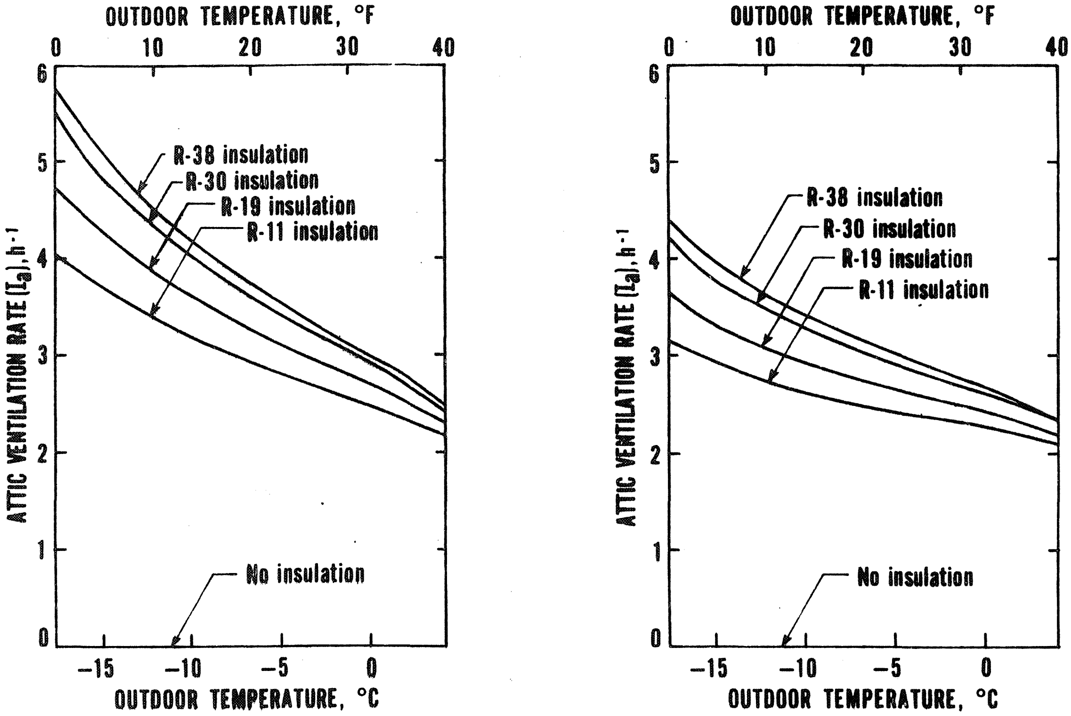
a) Without Vapor Barrier   *b) With Vapor Barrier*

Fig. 5 Required attic ventilation rates for a house without humidification [ceiling air penetration rate = 0 $m^3/s \cdot m^2$ (0 cfm/ft^2)]



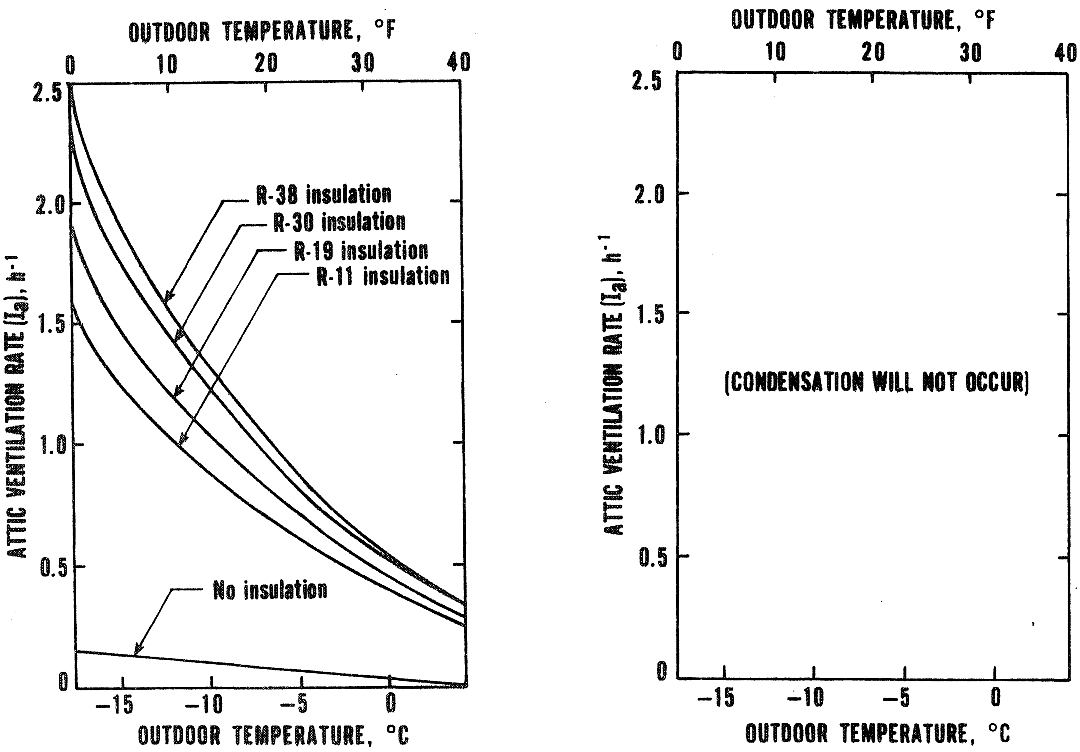
a) Without Vapor Barrier   *b) With Vapor Barrier*

Fig. 6 Required attic ventilation rates for a house without humidification [ceiling air penetration rate = 1.3 x 10^{-4} $m^3/s \cdot m^2$ (0.025 cfm/ft^2)]



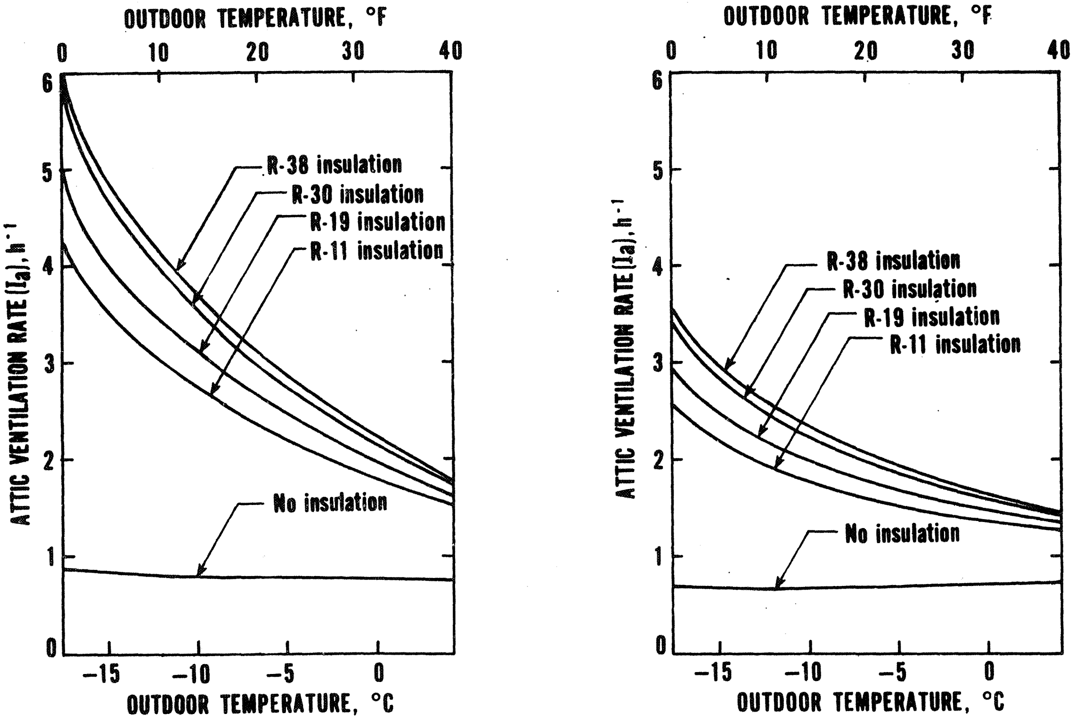
a) Without Vapor Barrier　　*b) With Vapor Barrier*

Fig. 7 Required attic ventilation rates for a house without humidification [ceiling air penetration rate = 2.5×10^{-4} $m^3/s \cdot m^2$ (0.05 cfm/ft^2)]



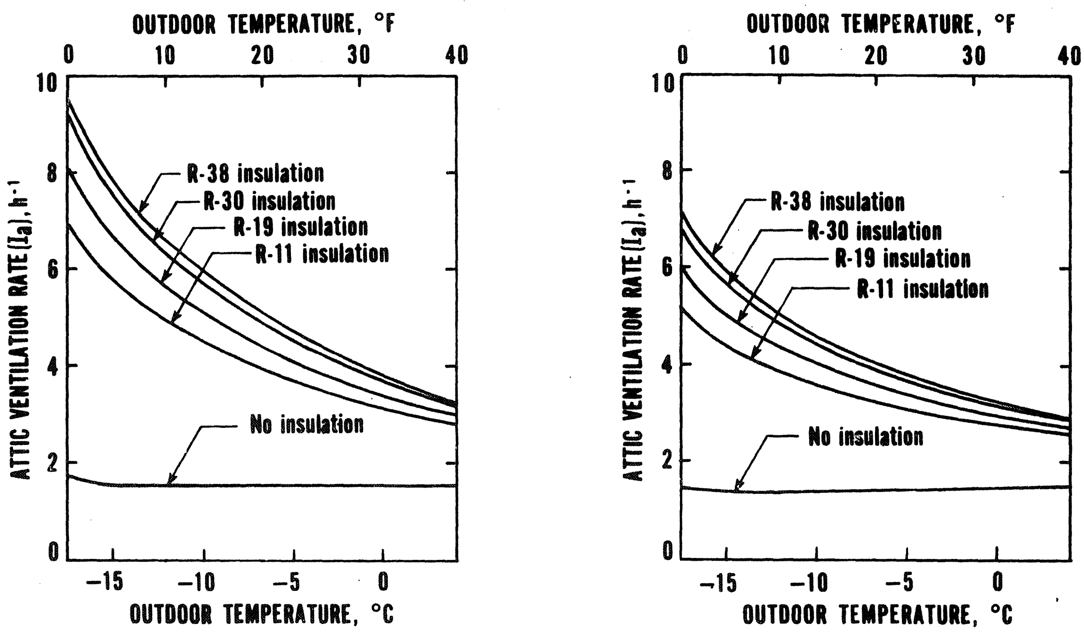
a) Without Vapor Barrier　　*b) With Vapor Barrier*

Fig. 8 Required attic ventilation rate for a house with humidification [ceiling air penetration rate = 0 $m^3/s \cdot m^2$ (0 cfm/ft^2)]



a) Without Vapor Barrier *b) With Vapor Barrier*

Fig. 9 Required attic ventilation rate for a house with humidification [ceiling air penetration rate = 1.3 x 10^{-4} $m^3/s \cdot m^2$ (0.025 cfm/ft^2)]



a) Without Vapor Barrier *b) With Vapor Barrier*

Fig. 10 Required attic ventilation rate for a house with humidification [ceiling air penetration rate = 2.5 x 10^{-4} $m^3/s \cdot m^2$ (0.050 cfm/ft^2)]