ENCORE-CANADA: COMPUTER PROGRAM FOR THE STUDY OF ENERGY CONSUMPTION OF RESIDENTIAL BUILDINGS IN CANADA

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ABSTRACT - The paper describes the mathematical methods employed in the ENCORE-CANADA computer program which predicts the hourly as well as the annual heating requirements of small residential-type buildings. ENCORE-CANADA is primarily intended for researchers, designers and consulting engineers interested in energy conservation measures. The model includes the effects of thermal storage, internal heat gains, basement and air infiltration losses, transmission heat losses and solar heat gains. The heating system is a thermostatically controlled oil-fired furnace with warm air distribution. Hourly solar radiation and weather data for various Canadian cities are used to simulate outdoor conditions.

RÉSUMÉ - Les auteurs décrivent les méthodes mathématiques utilisées dans le programme informatique ENCORE-CANADA qui prédit les besoins de chauffage horaires et annuels des petits immeubles résidentiels. ENCORE-CANADA s'adresse surtout aux chercheurs, aux concepteurs et aux ingénieurs-conseils qui s'intéressent aux mesures d'économie de l'énergie. Le modèle englobe les effets de l'inertie thermique, les gains de chaleur internes, les pertes de chaleur par les fondations et l'infiltration d'air, les pertes par transmission et les gains de chaleur solaire. L'installation de chauffage à air chaud pulsé utilise l'huile comme combustible et est réglée par thermostat. Des données météorologiques et le rayonnement solaire horaire en différents centres canadiens servent à simuler les conditions extérieures.

INTRODUCTION

The determination of the heat energy required to maintain prescribed indoor conditions in a building is one of the main concerns of building designers. (Ayres 1977). For many years the degree-day method (ASHRAE 1976) was widely used but, some of the essential elements of energy calculations are absent from such simple methods (Mitalas 1976). Owing to the relatively high cost of energy there is a need to refine the accuracy of building energy requirement calculations. The use of the computer is inevitable due to the length and complexity of computations for hour-by-hour analysis. Furthermore, it is important to predict accurately not only the total energy consumption of a building, but also the savings as a result of energy conservation measures. Thus the mathematical model includes many previously ignored factors.

This paper describes a mathematical model for hour-by-hour simulation of energy consumption of small residential-type buildings using real weather data. The associated computer program, ENCORE-CANADA (Energy Consumption of Residences in Canada) is the Canadian version of the Norwegian ENCORE program (Larsen 1976 and 1977). It allows one to study, for a given building design, the relation between heating demand and

- climatological factors (e.g. temperature, wind, solar radiation),
- building location and orientation,
- insulation levels in ceiling/roof system, walls and basement,
- thermal storage in the building structure and contents,
- types of fenestration,
- exterior surface solar absorptivity,
- air infiltration,
- thermostat settings, and
- heating system characteristics.

OVERVIEW OF MODEL

ENCORE-CANADA requires two input data sets: one describing the building model, and the other containing hourly weather and solar radiation data. The building model has the following components:

- basement with below- and above-grade portions,
- non-partitioned building enclosure that includes ceiling/roof system, walls, doors and windows,
- thermostatically controlled warm air distribution heating system consisting of oil-fired furnace, smoke pipe with barometric damper, and chimney,
- two-element electric hot water heater and storage tank from which water is drawn according to a 24 h schedule,
- lights which are on or off according to a 24 h schedule,
- various heat-generating electrical appliances and equipment that are turned on or off according to a 24 h schedule,
- occupants who are present in the building in varying numbers according to a 24 h schedule.

Schedules for holidays may be different from those for working days. Using the schedules and the specifications of model components, ENCORE-CANADA computes the internal heat gain due to

- occupants,
- electrical appliances and equipment,
- lights, and
- hot water consumption.

Similarly, the program uses weather and solar radiation data and the specifications of model components to compute heat gain and loss through

- below-grade portion of basement,
- ceiling/roof system and exterior walls including above grade portion of basement,
- doors,
- windows, and
- air infiltration.

These gains or losses, with the exception of the below-grade portion of the basement are computed with respect to an arbitrarily chosen indoor reference temperature. The sum of various heat gains and losses computed with respect to the reference temperature are transformed into heating demand based on indoor air temperature controlled by thermostat.

WEATHER AND SOLAR RADIATION DATA

ENCORE-CANADA may use any one of 39 preprocessed weather and solar radiation data sets. They correspond to three consecutive years of data for the following 13 Canadian cities.

Vancouver, B.C.	(1970-72)
Edmonton, Alta.	(1970-72)
Winnipeg, Man.	(1970-72)
Ottawa, Ont.	(1970-72)
Fredericton, N.B.	(1970-72)
Toronto, Ont.	(1970-72)
Montreal, Que.	(1970-72)
Halifax, N.S.	(1970-72)
St. John's, Nfld.	(1970-72)
Summerland, B.C.	(1970-72)
Suffield, Alta.	(1970-72)
Swift Current, Sask.	(1970-72)
Charlottetown, P.E.I.	(1972-74)

Each set contains the city's latitude (ℓ) and the following data for each day of the year:

- date (day, month),
- holiday indicator (used for selecting schedules),
- hour angle when solar altitude is zero (sunrise angle, h_o),
- tangent of declination angle, $\tan(\delta)$,
- ground reflectivity, R (0.4 with snowcover, 0.2 without).

For each day of the year the data set contains 24 hourly values of

a) dry-bulb temperature of ambient air (θ_o),

b) total cloud amount in tenths (C),

c) wind speed (V), 10 m above ground, at meteorological site,

d) wind direction,

e) atmospheric pressure (P),

f) hour angle (h), (sun is above horizon if $|h|<|h_o|$),

g) intensity of diffuse sky radiation (M_s),

h) intensity of direct solar radiation per unit area normal to sun's rays (M_{dn}),

i) conversion factor for items g) and h), (e).

Of these, a) through e) are measured or observed quantities; f) through i) are computed values. ENCORE-CANADA applies the following conversion formula to items g) and h)

$$I(0) = \left[\frac{100}{100-e(0)}\right]\left[\frac{M(0) + M(1)}{2}\right] \qquad (1)$$

where

$I(0)$ = average solar radiation during present hour,

$e(0)$ = percent difference between computed and measured average total solar radiation (direct plus diffuse) on a horizontal surface during present hour,

$M(0)$ = calculated instantaneous solar radiation at present hour,

$M(1)$ = calculated instantaneous solar radiation at previous hour.

The factor 100/[100-e(0)] in Eq. (1) is a cloud cover modifier derived by comparing computed and measured total solar radiation on a horizontal surface.

ENCORE-CANADA uses the information in the weather and solar radiation data sets to compute the solar radiation intercepted by building components. The number of distinct surface orientations for a given building envelope are determined in the subroutine SOLRAD. For every distinct surface orientation the subroutine DIRDIF calculates the direct and diffuse components of solar radiation per unit area according to ASHRAE procedure (ASHRAE 1975). The direct component is given by

$$I_{dir} = I_{dn}\cos(i)$$

$$= I_{dn}\{[\sin(\delta)\sin(\ell) + \cos(\delta)\cos(\ell)\cos(h)]\cos(q)$$

$$-[\sin(\delta)\cos(\ell) - \cos(\delta)\sin(\ell)\cos(h)]\sin(q)\cos(a)$$

$$-\cos(\delta)\sin(h)\sin(q)\sin(a)\} \qquad (2)$$

where

- i = incidence angle
- a = surface azimuth angle (measured counter-clockwise from south)
- q = surface tilt angle (measured from the horizontal)
- ℓ = latitude
- δ = declination angle
- h = hour angle.

The diffuse component of solar radiation is given by

$$\left.\begin{aligned} I_{dif} &= I_s && 0° \le q < 45° \\ I_{dif} &= Y\,I_s + \frac{1-\cos(q)}{2}\,I_g && 45° \le q \le 135° \\ I_{dif} &= I_g && 135° < q \le 180° \end{aligned}\right\} \qquad (3)$$

where the ratio of diffuse vertical sky radiation to diffuse horizontal radiation, Y, is obtained from Threlkeld (1962)

$$\begin{aligned} Y &= 0.45 && \cos(i) < -0.2 \\ Y &= 0.55 + 0.437\cos(i) + 0.313\cos^2(i) && \cos(i) \ge -0.2 \end{aligned} \qquad (4)$$

and the intensity of diffuse ground radiation is given by

$$I_g = R\{I_s + I_{dn}[\sin(\delta)\sin(\ell) + \cos(\delta)\cos(\ell)\cos(h)]\} \qquad (5)$$

The total radiation $I_t = (I_{dir} + I_{dif})$ is essential to the computation of surface sol-air temperature from the following equation (ASHRAE 1977):

$$\theta = \theta_o + \frac{1}{H}\left[\alpha I_t - E\,D\,\frac{1+\cos(q)}{2}\left(1 - \frac{C}{10}\right)\right] \qquad (6)$$

where

- α = surface solar absorptivity
- E = radiant interchange factor for a horizontal surface facing the sky (assumed to be 0.9, 0.5 and 0.1 for flat, suburban and urban building sites, respectively)
- D = difference between incident long-wave sky radiation and radiation emitted by a horizontal surface facing the sky (63.1 W/m^2 (20 Btu/hr ft^2) was used)
- H = coefficient of heat transfer by long-wave radiation and convection (assumed to be constant and equal to 17 W/m^2 K, 3 Btu/hr ft^2 R).

In addition to the thirty-nine data sets mentioned above, the mean (θ_m), amplitude (θ_a) and phase (p) of the main sinusoidal component for the ground surface sol-air temperature function (for the same thirteen cities and three consecutive years) are stored in the block data subroutine SOLBLK. θ_m, θ_a and p are used as input to the below grade basement and heat loss model. They were obtained from a Fourier analysis of hourly values of ground surface sol-air temperature computed from Eq. (6) by setting $\alpha = 1 - R$, $\cos(q) = 1$, $E = 0.9$, by using measured values for the total solar radiation incident on the ground surface, and by assuming a linear variation of H with wind speed (V)

$$\left.\begin{aligned} H &= 11.36 + 0.353V \ [\mathrm{W/m^2\,K}] && \text{if } V \le 48.3 \text{ km/h} \\ H &= 28.4 \ [\mathrm{W/m^2\,K}] && \text{if } V > 48.3 \text{ km/h} \end{aligned}\right\} \qquad (7)$$

CALCULATIONS OF BELOW-GRADE BASEMENT HEAT LOSS

The finite difference program TRUMP (Edwards 1968) was used to investigate the importance of variables affecting heat flow from the below-grade portion of basements. The following factors were found to be important:

- amount of wall below grade,
- mean thermal properties of concrete and soil,
- mean and fundamental sinusoidal component of ground surface sol-air temperature,
- depth and temperature of water table,
- extent and thermal resistance of insulation,
- perimeter and shape of basement.

Factors such as freezing and thawing, temperature dependence of thermal properties, snow cover, position of insulation, and diurnal and weekly transients were found to have a second order effect on calculated heat loss for a heating season. Since most of the important parameters are seldom known accurately, a simplified algorithm was selected rather than incorporating a finite difference or finite element approach. The heat flow from the basement was assumed to be the sum of

- steady-state heat flow to ground surface at mean annual sol-air temperature,
- periodic heat flow to ground surface with temperature fluctuation equal to fundamental sinusoidal component of sol-air temperature, and
- steady-state heat flow to water table at mean annual deep ground temperature.

The basement interior surface temperature was assumed constant throughout the heating season. The components for wall and floor were treated separately to simplify comparison with the TRUMP finite difference results. Thus, in ENCORE-CANADA the hourly values of heat flow through basement walls (Q_w) and floor (Q_f) are computed with the subroutine BTLOSS according to the following formulae

$$Q_w = G_{wg}(\theta_m - \theta_{in}) + G_{ww}(\theta_{wt} - \theta_{in}) + [G_{wgc}\cos(\frac{2\pi}{n}t + p) + G_{wgs}\sin(\frac{2\pi}{n}t + p)]\theta_a \quad (8)$$

$$Q_f = G_{fg}(\theta_m - \theta_{in}) + G_{fw}(\theta_{wt} - \theta_{in}) + [G_{fgc}\cos(\frac{2\pi}{n}t + p) + G_{fgs}\sin(\frac{2\pi}{n}t + p)]\theta_a \quad (9)$$

Where

- t = hour of the year ($0 \leq t \leq n$)
- n = number of hours in a year (n = 8760 for ordinary year, n = 8784 for leap year)
- θ_{wt} = water table temperature
- θ_{in} = constant basement interior surface temperature
- θ_m = mean annual ground surface sol-air temperature
- θ_a = amplitude of fundamental Fourier sine component of ground surface sol-air temperature function
- p = phase of fundamental Fourier sine component of ground surface sol-air temperature function
- G_{wg} = steady-state heat conductance between basement walls and ground surface
- G_{ww} = steady-state heat conductance between basement walls and water table
- G_{wgc} = cosine component of dynamic heat conductance between basement walls and ground surface
- G_{wgs} = sine component of dynamic heat conductance between basement walls and ground surface
- G_{fg} = steady-state heat conductance between basement floor and ground surface
- G_{fw} = steady-state heat conductance between basement floor and water table
- G_{fgc} = cosine component of dynamic heat conductance between basement floor and ground surface
- G_{fgs} = sine component of dynamic heat conductance between basement floor and ground surface.

The function $\theta(t) = \theta_m + \theta_a[\sin(2\pi/n + p)]$ describes the periodic boundary condition for temperature at the ground surface. The first two terms on the right hand sides of Eqs. (8) and (9) represent steady-state heat flow from the basement to the ground surface and to the water table, respectively. The remaining terms represent the dynamic heat flow between basement and ground surface. The dynamic heat flow between basement and water table is neglected. The conductances G_{wg}, G_{ww}, G_{wgc}, G_{wgs}, G_{fg}, G_{fw}, G_{fgc} and G_{fgs} are computed once for a particular basement in the subroutine BASMNT. In order to arrive at these eight conductances, a version of the method described by Boileau and Latta (1968) is used.

AIR INFILTRATION CALCULATIONS

The air infiltration calculations are based on the model described in detail by Konrad et al (1978). This model is limited to eight different building height-to-width and length-to-width ratios. The leakage opening of each building component (ceiling, walls, windows and doors) is represented by several holes equally spaced along the height of the component. If the indoor pressure at the ground level (P_1) is known, the mass flow rate of air (G_j) through the j-th hole can be computed from

$$G_j = s\,\rho(R_j)^{-n_j}\left| P + \frac{P}{R}\left[\frac{c_j V^2}{2\theta_o} + g\,h_j\left(\frac{1}{\theta_i} - \frac{1}{\theta_o}\right)\right] - P_1\right|^{n_j} \quad (10)$$

where

- s = + 1 for air infiltration, and
  = - 1 for air exfiltration
- ρ = density of indoor air if s = - 1, and
  = density of outdoor air if s = + 1
- R_j = resistance to air leakage of j-th hole
- n_j = flow exponent for air flow through j-th hole
- P = outdoor atmospheric pressure at ground level
- R = gas constant for air
- c_j = wind pressure coefficient where j-th hole is located (depends on wind direction and building orientation)
- V = wind speed

g = gravitational acceleration

h_j = height of j-th hole above ground

θ_o = outdoor air temperature

θ_i = indoor air temperature.

Using the mathematical model of the furnace, smoke pipe, barometric damper and chimney outlined by Konrad et al (1978), the program (subroutines FRNACE, INFILT, FLOWS, PCOEF, WIND, CHMNEY, FFACT and PRESSR) computes the mass flow rate of gases in the chimney (G_c) as a function of furnace load factor for any assumed value of the indoor pressure at ground level. The true pressure is computed by an iterative procedure from the mass balance equation for air flow through m holes and the gas flow in the chimney, i.e.,

$$\sum_{j=1}^{m} G_j - G_c = 0 \tag{11}$$

The air infiltration load, L_i, varies with outdoor air temperature, indoor air temperature, wind speed, wind direction and furnace operation. It is computed by multiplying the sum of all positive G_j's by $(\theta_i - \theta_o)$ and by the specific heat of air at temperature $(\theta_i + \theta_o)/2$.

HEAT TRANSFER CALCULATIONS

The z-transform method

ENCORE-CANADA employs the z-transform method to compute:

- heat transferred through ceiling, walls, windows and doors,
- heat gained by indoor air from occupants, lights, domestic hot water and electrical appliances, and
- heating demand and room air temperature including effects of thermostat control system.

The z-transform method is closely related to the classical Laplace transformation. The method is perhaps easiest to understand in the context of sampled data feedback control systems for which it was originally developed (Truxal 1955). Its application to heat transfer problems in buildings is described by Mitalas (1978).

When a continuous function of time, i(t), is sampled at regular time intervals, T, the resulting function, i'(t), can be described in terms of the unit impulse function, $u_o(t)$, as follows:

$$i'(t) = \sum_{n=0}^{\infty} i(nT)\, u_o(t - nT) \tag{12}$$

The z-transform of i(t) is obtained from the Laplace transform of i'(t) by substituting z for exp(Ts). It is given by

$$I(z) = \sum_{n=0}^{\infty} i(nT) z^{-n} \tag{13}$$

If both input, i(t), and output, o(t), of a linear, invariable system are expressed in terms of their z-transforms, I(z) and O(z) respectively, then the ratio K(z) = O(z)/I(z) is a z-transform function for the system. If K(z) is known, the response to any given input is obtained by forming the product K(z) I(z). To determine K(z), one finds the impulse response of the system expressed as a polynomial in z, as follows:

$$K(z) = \sum_{n=0}^{p} k(nT) z^{-n} \tag{14}$$

where the coefficients, k(nT), are called response factors and $p = \infty$. If the response decays to zero with time, p can be assigned a finite value by truncating the polynomial. In practice, it is sometimes impossible to find the impulse response since it is associated with infinite heat flux. In such cases a triangular pulse of unit area is used as input to find the response factors (Kimura 1977; Stephenson and Mitalas 1967). K(z) can always be transformed into the ratio of two polynomials (Mitalas 1978) simply by multiplying Eq. (14) by a polynomial quotient which evaluates to 1, i.e.,

$$K(z) = \sum_{n=0}^{p} k(nT) z^{-n} \frac{\prod_{m=1}^{j} (1 - w_m z^{-1})}{\prod_{m=1}^{j} (1 - w_m z^{-1})} = \frac{\sum_{n=0}^{p+j} a_n z^{-n}}{\sum_{n=0}^{j} b_n z^{-n}} \tag{15}$$

It is often convenient for simple computations to set j = 1 and write K(z) in the form

$$K(z) = \sum_{n=0}^{p} k(nT) z^{-n} \frac{1 - w_1 z^{-1}}{1 - w_1 z^{-1}} = \frac{\sum_{n=o}^{p+1} v_n z^{-n}}{1 - w_1 z^{-1}} \tag{16}$$

where w_1 is called Common Ratio and is defined by

$$w_1 \doteq \lim_{n \to \infty} \frac{k(nT)}{k[(n-1)T]} \tag{17}$$

Whether given in the form of Eqs. (14), (15) or (16), in general, K(z) can always be thought of as the ratio of two polynomials, i.e., $K(z)=N(z)/D(z)$. Given the coefficients N_i and D_i of the numerator and denominator, respectively, together with a series of sampled values i(nT) and o(nT) of an excitation and a response respectively, except for the current value of the response o(qT), the latter can be solved by equating the coefficients of the highest power of z on both sides of the equation:

$$O(z)D(z) = I(z)N(z) \tag{18}$$

If N(z) has p+1 coefficients and D(z) has q+1 coefficients, one obtains

$$o(qT) = \frac{1}{D_0}\{i(0)N_p + i(T)N_{p-1} + \ldots + i[(p-1)T]N_1 + + i(pT)N_0 - o(0)D_q - o(T)D_{q-1} - \ldots - - o[(q-1)T]D_1\} \tag{19}$$

Heat transfer through walls and ceiling

A typical wall construction is shown in Fig. 1. The inner and outer layers of air and the airspace are assumed to have a thermal resistance (R_1 and R_6) but no thermal capacity. The other layers are specified by their thickness (T), thermal conductivity (K), density (D) and specific heat (S). There is no restriction on the number of layers. The sol-air temperature, $\theta(t)$, at the exterior surface is given by Eq. (6). The heat flow, Q(t), at the inside surface of the wall is related to the temperature difference $\theta(t) - \theta_r$ by a z-transfer function. A method to obtain the transfer function as a ratio of two polynomials in z is described by Stephenson and Mitalas (1971) and Mitalas (1968). ENCORE-CANADA requires n coefficients, a_i and b_i, (ASHRAE 1977; Mitalas and Arseneault 1972) to compute the heat flow during the present hour, Q(0), given the heat flows during previous hours (Q(i), $i \geq 0$) and the outdoor temperature during present and previous hours ($\theta(i)$, $i \geq 0$). The number of previous hours for which data are required depends on the number of coefficients (24 is the upper limit in ENCORE-CANADA). n is large for massive walls, small for light walls. On the basis of Eq. (19), Q(0) is computed from

$$Q(0) = \sum_{i=0}^{n} a_i A(\theta(i) - \theta_r) - \sum_{i=1}^{n} b_i Q(i) \tag{20}$$

where A is the net wall area. The coefficients b_i are dimensionless; the coefficients a_i are in watts per square metre degree Kelvin (British thermal units per hour square foot degree Rankine). The a_i must be computed under the assumption that $H = 1/R_1 = 17\ W/m^2\ K$ (3 Btu/hr ft^2 R).

The heat Q(0) which appears on the inside wall surface during the current hour, does not have an immediate effect on the indoor air temperature. The delay depends on the type of interior finish, furnishings and construction. It is possible to derive z-transfer functions for this system in the form given by Eq. (16) and as a function of mass per unit floor area (ASHRAE 1977; Mitalas 1972). The following coefficients are stored in the subroutine WALLGL for four different values of mass per unit floor area:

	146 kg/m² (30 lb/ft²)	342 kg/m² (70 lb/ft²)	635 kg/m² (130 lb/ft²)	976 kg/m² (200 lb/ft²)
v_0	0.703	0.681	0.676	0.676
v_1	-0.523	-0.551	-0.606	-0.646
w_1	-0.82	-0.87	-0.93	-0.97

Note that $v_1 = 1 + w_1 - v_0$. Based on Eq. (19), the amount of heat transferred to or removed from the indoor air by a wall surface during the present hour is computed in the subroutine WALLGL according to

$$L_s(0) = v_0Q(0) + v_1Q(1) - w_1L_s(1) \tag{21}$$

Heat transfer through doors

The heat flow through doors is calculated assuming steady-state conditions. The steady-state heat flow through a door of area A_d and thermal conductance U_d due to the difference between exterior sol-air temperature, $\theta(0)$, and indoor reference temperature, θ_r, is given by:

$$Q_d(0) = U_dA_d[\theta(0) - \theta_r] \tag{22}$$

Note that U_d must be given for the H used in Eq. (6). The heat transferred to or removed from the indoor air by a door surface during the present hour is computed in the subroutine DOORGL from an equation similar to (21)

$$L_d(0) = v_0Q_d(0) + v_1Q_d(1) - w_1L_d(1) \tag{23}$$

where the transfer function coefficients v_0, v_1 and w_1 are identical to those for walls.

Heat transfer through windows

Heat transfer through windows occurs by the combined mechanism of long-wave radiation, conduction, convection and by transmission and absorption of solar radiation. Consider a window glazing system with or without interior shading. The solar heat gain through such a window during the present hour can be computed from the solar heat gain factor, *SHGF*(0),

$$Q_s(0) = A_wS_w SHGF(0) = A_wS_w\{I_{dif}(0)[t_{dif} + \alpha_{dif} F] + + I_{dir}(0)[t_{dir} + \alpha_{dir} F]\} \tag{24}$$

where

A_w = window surface area

S_w = window shading coefficient (ratio of solar heat gain through glazing system to solar heat gain through double-strength single pane window glass under identical climatic conditions)

I_{dif} = intensity of diffuse solar radiation from Eq. (3)

I_{dir} = intensity of direct solar radiation from Eq. (2)

t_{dif} = 0.8 = transmissivity for diffuse radiation of single 0.3175 cm (1/8 in) thick ordinary window pane (Mitalas and Stephenson 1962; Stephenson 1965)

t_{dir} = transmissivity for direct radiation of single window pane

α_{dif} = 0.054 = absorptivity for diffuse radiation of single window pane (Mitalas and Stephenson 1962; Stephenson 1965)

α_{dir} = absorptivity for direct radiation of single window pane

F = inward-flowing fraction of absorbed solar radiation for single window pane (given by ratio of outside surface thermal resistance to total resistance; estimated to be 1/3)

The transmissivity and absorptivity for direct solar radiation are functions of the cosine of the incidence angle (ASHRAE 1977; Stephenson 1965)

$$t_{dir} = -0.00885 + 2.71235\cos(i) - 0.62062\cos^2(i) - 7.07329\cos^3(i) + 9.75995\cos^4(i) - 3.89922\cos^5(i)$$

$$(\text{if } t_{dir} < 0.0, \text{ set } t_{dir} = 0.0) \quad (25)$$

$$\alpha_{dir} = 0.01154 + 0.77674\cos(i) - 3.94657\cos^2(i) + 8.57881\cos^3(i) - 8.38135\cos^4(i) + 3.01188\cos^5(i) \quad (26)$$

The heat transferred through a window of thermal conductance U_w due to indoor/outdoor temperature difference is given by an equation similar to (22) for doors

$$Q_c(0) = U_w A_w[\theta(0) - \theta_r] \quad (27)$$

where the window exterior surface temperature, $\theta(0)$, is computed from Eq. (6) with αI_t set to zero (solar radiation is already accounted for in Eq. (24)). Note that U_w must be given for the H used in Eq. (6).

Given the values of Q_s and Q_c for the present and previous hour, the heat transferred to the indoor air from a window surface is obtained by the transfer function method as

$$L_w(0) = s_0 Q_s(0) + s_1 Q_s(1) + v_0 Q_c(0) + v_1 Q_c(1) - w_1 L_w(1) \quad (28)$$

$L_w(0)$ is computed in the subroutine WNDWGL. The transfer function coefficients v_0, v_1 and w_1 are identical to those for wall surfaces. The s_0 and s_1 coefficients for a window without shading (i.e. no blinds, curtains or drapes) are stored in the subroutine WNDWGL for four values of mass per unit floor area

	146 kg/m² (30 lb/ft²)	342 kg/m² (70 lb/ft²)	635 kg/m² (130 lb/ft²)	976 kg/m² (200 lb/ft²)
s_0	0.224	0.197	0.187	0.187
s_1	-0.044	-0.067	-0.117	-0.157

When the window has interior shading (blinds, curtains or drapes), $s_0 = v_0$ and $s_1 = v_1$, and the values of S_w and U_w are changed. When the window has a closed shutter on the exterior, $S_w = s_0 = 0.0$, the thermal conductance, U_w, is changed and the exterior surface sol-air temperature, $\theta(0)$, is computed from Eq. (6) as for wall surfaces (i.e. with αI_t included).

Internal heat gain due to lights

The heat transferred to indoor air by lights represents a significant portion of the electrical power supplied since electric lights have fairly low efficacy. Some of the energy is dissipated by convection, and a significant part by radiation (Kimura 1977; Mitalas 1973; and 1973-74). The transfer function method yields the following formula for the computation of convective and radiant heat gain from lights:

$$L_u(0) = J_1 y_1 Q_u(1) + J_2 y_2 Q_u(2) - w_1 L_u(1) \quad (29)$$

where $Q_u(1)$ and $Q_u(2)$ are the electrical energy input to lights one and two hours ago, respectively. The values for Q_u are given in a schedule in the ENCORE-CANADA input data. Notice that $Q_u(0)$ does not appear on the right hand side of Eq. (29), indicating a long delay between the energy input to lights during the present hour and the consequent rise in indoor air temperature. $L_u(0)$ is computed in the subroutine IGAIN2, where the z-transfer function coefficients, y_1 and y_2, are stored for four different values of mass per unit floor area

	146 kg/m² (30 lb/ft²)	342 kg/m² (70 lb/ft²)	635 kg/m² (130 lb/ft²)	976 kg/m² (200 lb/ft²)
y_1	0.5	0.5	0.5	0.5
y_2	-0.32	-0.37	-0.43	-0.47

The w_1 coefficients are identical to those used for heat transfer through walls, doors and windows.

The coefficients J_1 and J_2 in Eq. (29) are normally set equal to 1 for lights that are turned on or off strictly according to a given schedule. However, lights may also be turned on or off according to the availability of sunlight. Thus, $J_2 = 0$ if there was sunlight two hours ago; $J_1 = 0$ if there was sunlight one hour ago.

Internal heat gain due to occupancy and appliances

The heat generated by occupants, Q_o, is transferred to the surroundings partly by radiation, partly by convection. The heat transferred by convection produces an almost instantaneous rise in surrounding air temperature. If the fraction of heat transferred by radiation is r (0.5 is a reasonable value), the fraction transferred by convection is

$$L_{oc}(0) = (1 - r)Q_o(0) \tag{30}$$

The gain due to radiant heat transfer from occupants for the present hour is given by an equation similar to (29) for lights

$$L_{or}(0) = r[u_1Q_o(1) + u_2Q_o(2)] - w_1L_{or}(1) \tag{31}$$

Notice the one hour delay between excitation and response. By adding Eqs. (30) and (31) one obtains the total heat gain due to occupancy as

$$L_o(0) = (1 - r)Q_o(0) + r[u_1Q_o(1) + u_2Q_o(2)] - w_1L_{or}(1) \tag{32}$$

The radiant component of heat gain during previous hour is simply

$$L_{or}(1) = L_o(1) - L_{oc}(1) = L_o(1) - (1 - r)Q_o(1) \tag{33}$$

and hence, by substituting in Eq. (32), one obtains

$$L_o(0) = (1 - r)Q_o(0) + [w_1(1 - r) + ru_1]Q_o(1) + ru_2Q_o(2) - w_1L_o(1) \tag{34}$$

The transfer function coefficient u_1 is identical to s_0 and u_2 is identical to s_1 for heat transfer through windows. $L_o(0)$ is computed by the subroutine IGAIN1.

Heat gain from electrical appliances and equipment, $L_e(0)$, is treated in the same manner as heat gain from occupants.

Heating from domestic hot water system

Most small residential-type buildings in Canada have an electric hot water heater which consists of a tank of volume V and an upper and lower heater element with maximum output of E_1 and E_2, respectively. The incoming cold water is at temperature θ_{in} and the hot water leaving the tank is at temperature θ_{out}. The maximum amount of heat the water in a full tank can store is Q_s and is given by

$$Q_s = \rho V C_p (\theta_{out} - \theta_{in}) \tag{35}$$

where ρ is the density and C_p is the specific heat of water at temperature $(\theta_{out} + \theta_{in})/2$. If the volume of hot water demand during the present hour is v(0), the amount of heat, $q_d(0)$, required to raise its temperature from θ_{in} to θ_{out} is $\rho v(0) \times C_p (\theta_{out} - \theta_{in})$. The make-up heat required is given by

$$q_m(0) = Q_s - q_s(1) + q_d(0) + q_\ell(0) \tag{36}$$

where $q_s(1)$ is the amount of heat left from the previous hour and $q_\ell(0)$ represents the standby loss from the tank (CSA 1973) and the distribution lines (Schultz and Goldschmidt 1978).

If $E_1 \geq q_m(0)$, heater element No. 1 produces $e(0) = q_m(0)$ amount of heat and the tank will be completely filled with hot water, i.e., $q_s(0) = Q_s$.

If $E_1 < q_m(0)$ and $E_2 \geq q_m(0)$, heater No. 1 turns off, and heater No. 2 will meet the demand (i.e., $q_s(0) = Q_s$) by generating $e(0) = q_m(0)$ amount of heat. If $E_2 < q_m(0)$, the demand cannot be met for this hour and only $e(0) = E_2$ amount of heat is generated, i.e., $q_s(0) = Q_s - q_m(0) + E_2$.

This procedure is implemented in the subroutine HOTWTR to arrive at the total electrical energy, e(0), consumed by the hot water system during the present hour. It is assumed that $Q_h(0) = [e(0) - q_\ell(0)]/2 + q_\ell(0)$ portion of e(0) causes the indoor air temperature to rise. The heat transfer mechanism is assumed to be identical to that for occupants and electrical appliances. Thus, the internal heat gain, $L_h(0)$, from the domestic hot water system is computed from Eq. (34) (Q_h replaces Q_o) by the subroutine IGAIN1.

THERMOSTAT CONTROL SYSTEM

The net heat loss or heat gain during the present hour, $L_n(0)$, is given by

$$L_n(0) = Q_w(0) + Q_f(0) + L_i(0) + L_s(0) + L_d(0) + L_w(0) + L_u(0) + L_o(0) + L_e(0) + L_h(0) \tag{37}$$

where the components are

- Q_w : basement floor, Eq. (8)
- Q_f : basement walls, Eq. (9)
- L_i : air infiltration
- L_s : walls and ceiling/roof system, Eq. (21)
- L_d : doors, Eq. (23)
- L_w : windows, Eq. (28)
- L_u : lights, Eq. (29)
- L_o : occupants, Eq. (34)
- L_e : electrical appliances

L_h : hot water system.

The net effect of indoor temperature variation during a heating season on basement heat losses (Q_w and Q_f) is neglected in ENCORE-CANADA. The components L_u, L_o, L_e and L_h can be considered independent of indoor temperature variations. However, the components L_i, L_s, L_d and L_w, which are computed with respect to a constant indoor reference temperature, θ_r, are functions of indoor/outdoor temperature difference. Therefore the effect of indoor temperature variation cannot be neglected.

A z-transfer function which relates the deviation of indoor air temperature, θ_i, from the reference temperature, θ_r, to the net amount of heat added to or removed from the indoor air can be derived from heat balance considerations and the thermal characteristics of the building enclosure (ASHRAE 1977; Mitalas 1972). The transfer function has the form of Eq. (16) with p = 1, and the coefficient w_1 is identical to the one used in Eqs. (21), (23), (28), (29) and (34). The coefficients v_0, v_1 and v_2 are calculated from

$$v_0 = g_0 A + w_0[K_{as} + I(0)] \qquad (w_0 = 1) \qquad (38)$$

$$v_1 = g_1 A + w_1[K_{as} + I(1)] \qquad (39)$$

$$v_2 = g_2 A + w_2[K_{as} + I(2)] \qquad (w_2 = 0) \qquad (40)$$

where

g_i = normalized room air transfer function coefficients (i = 0, 1, 2)

A = floor surface area

K_{as} = total thermal conductance between indoor air and surroundings (computed in the subroutine FCFACT)

$I(j)$ = air infiltration heating load per unit indoor/outdoor temperature difference, j hours ago.

The normalized room air transfer function coefficients, g_0, g_1 and g_2, are stored in the subroutine DEMAND for four different values of mass per unit floor area

	146 kg/m² (30 lb/ft²)	342 kg/m² (70 lb/ft²)	635 kg/m² (130 lb/ft²)	976 kg/m² (200 lb/ft²)	
g_0	9.54	10.28	10.50	10.50	[W/m² K]
	1.68	1.81	1.85	1.85	[Btu/hr ft² R]
g_1	-9.82	-10.73	-11.07	-14.19	[W/m² K]
	-1.73	-1.89	-1.95	-2.50	[Btu/hr ft² R]
g_2	0.28	0.45	0.57	3.69	[W/m² K]
	0.05	0.08	0.10	0.65	[Btu/hr ft² R]

They were derived for unit floor area under the assumption of negligible air infiltration and conductance between indoor air and exterior surroundings. Equations (38) to (40) adjust the normalized coefficients for a specific situation (ASHRAE 1977; Mitalas 1972).

Let the excitation be the deviation of indoor air temperature from the reference temperature, $\theta_i(t) - \theta_r$, and let the response be the algebraic sum of the heat demand, E(t), and the net heat loss or heat gain, $L_n(t)$. Application of Eq. (19) yields a relationship between heat demand and indoor air temperature

$$E(0) + L_n(0) = v_0[\theta_i(0) - \theta_r] + v_1[\theta_i(1) - \theta_r] + v_2[\theta_i(0) - \theta_r] - w_1[E(1) + L_n(1)] \qquad (41)$$

By denoting

$$Z(0) = -(v_0 + v_1 + v_2)\theta_r + v_1\theta_i(1) + v_2\theta_i(2) - w_1[E(1) + L_n(1)] - L_n(0) \qquad (42)$$

one can rewrite Eq. (41) as

$$E(0) = v_0\theta_i(0) + Z(0) \qquad (43)$$

Since both E(0) and $\theta_i(0)$ are unknown, another independent relationship between heat demand and indoor temperature during the present hour is required. This is provided by the mathematical model of the thermostat control system.

A simple thermostat, which is set at temperature θ_s, functions as shown in Fig. 2 where the heat output, E, of the heating system is plotted against the indoor air temperature, θ_i. A band, θ_b, about the setpoint, θ_s, determines when the heating system is turned on or off.

When the temperature falls below $\theta_s - \theta_b/2$, the heating system turns on; when the temperature rises above $\theta_s + \theta_b/2$, the heating system turns off. During a 1 h period the thermostat may go through several on/off cycles. Consequently, the on/off cycles of a thermostat control system cannot be simulated in hour-by-hour calculations. Instead, the simplified representation shown in Fig. 3 is used. It is based on the observation that within the thermostat band, θ_b, the total amount of heat produced is proportional to the indoor air temperature.

Mathematically, the proportional band thermostat control system is represented as follows:

$$E(0) = E_{max} \qquad \theta_i(0) < \theta_s(0) - \tfrac{1}{2}\theta_b \qquad (44a)$$

$$E(0) = S\,\theta_i(0) + E_0(0)$$

$$\theta_s(0) - \tfrac{1}{2}\theta_b \leq \theta_i(0) \leq \theta_s(0) + \tfrac{1}{2}\theta_b \qquad (44b)$$

$$E(0) = 0.0 \qquad \theta_i(0) > \theta_s(0) + \tfrac{1}{2}\theta_b \qquad (44c)$$

where the slope, S, and the vertical intercept, $E_0(0)$, are given by (see Fig. 3)

$$S = -\frac{E_{max}}{\theta_b} \qquad (45)$$

and

$$E_0(0) = \tfrac{1}{2} E_{max} - S\,\theta_s(0) \qquad (46)$$

respectively. Equations (43) and (44a, b, c) can be solved simultaneously for $\theta_i(0)$ and $E(0)$ to give

$$\left.\begin{aligned} E(0) &= E_{max} \\ \theta_i(0) &= \frac{E_{max} - Z(0)}{v_0} \end{aligned}\right\} \theta_i(0) < \theta_s(0) - \tfrac{1}{2}\theta_b \qquad (47a)$$

$$\left.\begin{aligned} E(0) &= \frac{S\,Z(0) - v_0 E_0(0)}{S - v_0} \\ \theta_i(0) &= \frac{Z(0) - E_0(0)}{S - v_0} \end{aligned}\right\} \theta_s(0) - \tfrac{1}{2}\theta_b \leq \theta_i(0) \leq \theta_s(0) + \tfrac{1}{2}\theta_b \qquad (47b)$$

$$\left.\begin{aligned} E(0) &= 0.0 \\ \theta_i(0) &= \frac{-Z(0)}{v_0} \end{aligned}\right\} \theta_i(0) > \theta_s(0) + \tfrac{1}{2}\theta_b \qquad (47c)$$

The indoor air temperature, $\theta_i(0)$, and the heat supplied by the heating system during the present hour, E(0), are computed according to Eqs. (47a, b, c) in the subroutine DEMAND. Note that θ_i, θ_s, Z, E_0 and E vary with time. The thermostat settings, θ_s, are specified for every hour of the day by a schedule in the ENCORE-CANADA input data.

CONCLUSIONS

The set of mathematical procedures used by the ENCORE-CANADA computer program has been described. Trial runs with ENCORE-CANADA produced results that exhibit expected behaviour. Such computational tests, however, can only prove the relative validity of the program results. Absolute validity remains to be determined by comparison with measurements. Collection of data from houses located in Ottawa is currently being undertaken by the Division of Building Research of the National Research Council of Canada.

The usefulness of the results of a computer simulation of building energy consumption is directly related to the limitations of the mathematical model and the accuracy of the available input data. More often than not, building science relies on crude estimates of the properties of building material and environmental conditions. In addition, complicated stochastic processes such as weather, solar radiation, sol-air temperature and internal building environment, are difficult to model without simplifying assumptions. When an assumption proves to be inappropriate, a more detailed, possibly more complicated, mathematical model has to be adopted.

ENCORE-CANADA was written with the intent to study as many as possible of the factors that influence energy consumption in small residential-type buildings. This necessitated a detailed, numerically stable model and, consequently, a lengthy computer program. The amount of arithmetic computing time required per simulation of a heating season is at present of the order of one hour on an IBM 360/67 computer, with about 5/6th of the time being spent by infiltration calculations. It is expected that if more efficient numerical methods are adopted for infiltration calculations, the computing time for one simulation would be 10 to 15 minutes.

The derivations given are based on an oil-fired furnace heating system with warm air distribution; the building is assumed to have no internal partitions. ENCORE-CANADA, however, can also simulate electrically heated, partitioned buildings. In this case many of the program algorithms are simplified and computing times are reduced.

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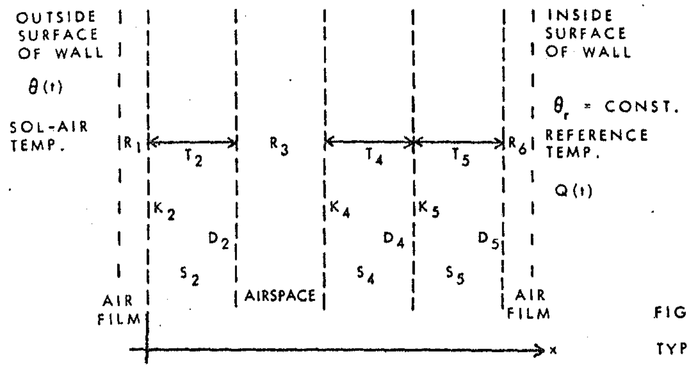

FIGURE 1

TYPICAL WALL CONSTRUCTION

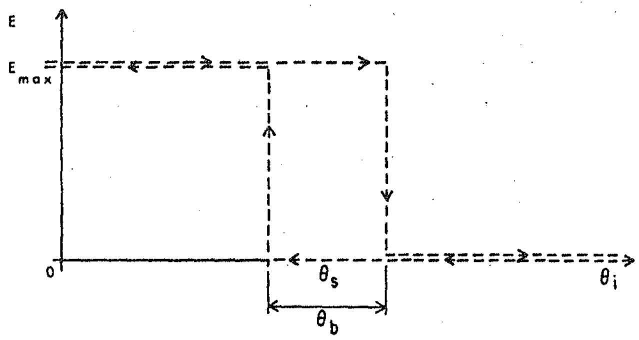

FIGURE 2

ILLUSTRATION OF A SIMPLE THERMOSTAT CONTROL ACTION (NOT TO SCALE)

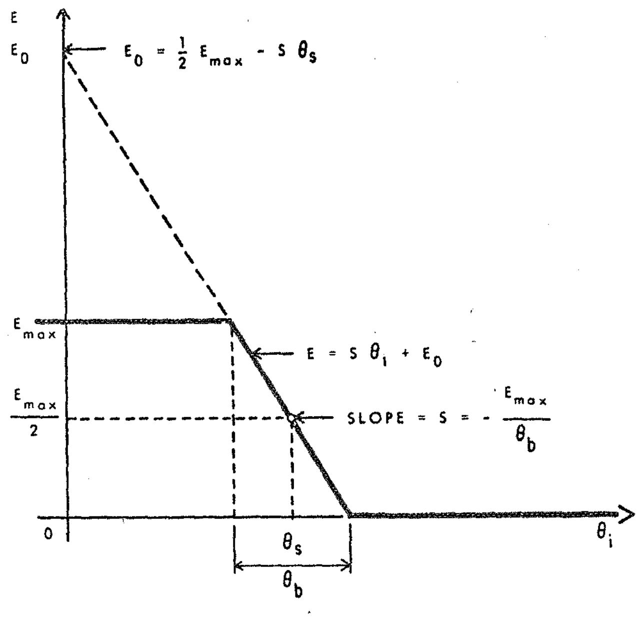

FIGURE 3

PROPORTIONAL BAND THERMOSTAT CONTROL ACTION (NOT TO SCALE)