INFILTRATION-PRESSURIZATION CORRELATIONS: SURFACE PRESSURES AND TERRAIN EFFECTS

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INTRODUCTION

In the past, the main concern for studying air flow around buildings has been to describe wind loads for design purposes, to locate intake and exhaust ports for HVAC systems, and to insure adequate stack effect for combustion processes. Recently, designing for energy conservation has underlined the importance in understanding how air flow around buildings affects heat loss through infiltration.

The energy consumption due to air infiltration can be a significant amount (20-40%) of the energy consumed in single-family dwellings. Reduction of this energy use will come through improved design, improved construction practices, and through improvements in existing buildings. In this paper we look at the effect of air flow around buildings on air infiltration for residential construction.

Two general procedures exist to test for infiltration--pressurization air leakage (both AC and DC types) and tracer gas concentration measurements. Several research groups are attempting to determine if it is possible to find a correlation between air leakage rates using fan pressurization and infiltration rates measured using a tracer gas.

Many reasons motivate the search for a correlation between the two procedures. The most important is the relative simplicity of the pressurization technique when compared to an infiltration measurement made with a tracer gas. In the pressurization technique a pressure difference is achieved by temporarily sealing a fan to the building shell. Mass flow continuity then predicts that the air flow rate through the fan is equal to the air leakage rate of the structure at the working pressure difference. A fan pressurization measurement of air leakage is not an analog of natural infiltration because the pressure distributions on the building shell are quite different in the two cases. In the first, the pressure over the entire shell is quite uniform while in the second, the pressures vary in a complicated manner both in space and in time. But measurements of air leakage in two structures using the same working pressure does

allow one to compare the two structures. If this comparison can also be used to predict comparative air infiltration rates for the two structures then the fan pressurization measurement will be very useful, indeed.

We are currently attempting to find a correlation by investigating the relationship between infiltration rates and surface pressures on a building. Measurement of the air leakage of the building shell as a function of pressure yields an average leakage function. Knowledge of the average surface pressure on the shell and the leakage function permits calculation of the natural infiltration rate. While measurements of the surface pressures are useful for developing and refining a model of infiltration, they are impractical on a large scale; surface pressures must be determined by calculation.

In this paper we develop expressions to calculate the surface pressure when wind speeds and indoor-outdoor temperature differences are known. We identify the neutral pressure level, the height at which the indoor and outdoor pressures are equal, as an important parameter for characterizing surface pressures. Tamura[1] has previously discussed the importance of the height of the neutral pressure level in determining natural infiltration rates. Our analysis extends his results and shows that the neutral pressure level depends upon wind speed and direction, and will therefore be different for each face of the building. The neutral pressure level can be measured in the field (a procedure is described below) or estimated from weather and terrain parameters combined with a measurement of the neutral level in a no wind condition.

In order to calculate the infiltration from known surface pressures we use the model of Grimsrud, et al[2]. In this model the infiltration is proportional to the average positive surface pressure and the exfiltration is proportional to the average negative surface pressure. The proportionality constant is the leakage function.

$$Q^+ = \sum_{j=0}^{5} A_j l_j \Delta P_j^+ \qquad\qquad 1$$

$$Q^- = \sum_{j=0}^{5} A_j l_j \Delta P_j^- \qquad\qquad 2$$

The index j denotes the six faces of the structure with j=0 being the floor, j=1,2,3,4 for the walls and j=5 represents the ceiling. (See the symbol table for the list of definitions.)

If we assume that the leakage through each face is the same, then the equations simplify.

$$Q^{\pm} = L \sum_{j=0}^{5} \Delta P_j^{\pm} \frac{A_j}{A} \qquad\qquad 3$$

where   L is the leakage constant derived from fan pressurization measurements.

VENTS

Treating air as an incompressible gas, flow continuity would predict that Q^+ would equal Q^-. The above analysis does not take into account the action of vents or other large openings. A large opening will not respond to the average pressure difference between the inside and one of the faces, but rather it will respond to the specific pressure drop in a way much different than Eq 3.

In order to take this into account the assumption is made that the flow through all the vents is unidirectional, that is, the vents will have air either going in or coming out; we assume that at any one time it does one or the other. In general this is a reasonable assumption since vents tend to be placed in or near the ceiling of a structure. Since flow through vents is not explicitly included in Q^+ or Q^-, the presence of vents will remove the constraint that Q^+ be equal to Q^-. This leads to the condition,

$$\Delta Q = Q^+ - Q^- ,$$ (4)

where ΔQ is the air flow through vents.

Actual infiltration is the larger of Q^+ and Q^-.

$$Q = MAX(Q^+, Q^-) .$$ (5)

Grimsrud, et al. (3) compared predicted infiltration from the above model with measured infiltrations. Several houses in different climates were tested and the results grouped, yielding the relationship

$$\frac{actual\ infiltration}{predicted\ infiltration} = 1.35 \pm .58 .$$ (6)

The scatter in results was, in part, due to the insensitivity of the measurement procedure to stack effect pressures; pressures resulting from density differences could not be measured.

STACK EFFECT

The calculation of the stack effect pressures is derived for the general case in Appendix A and outlined briefly below. The stack effect is a pressure difference caused by a temperature difference between inside and out. If we assume an isothermal atmosphere and expand the expression for the pressure as a function of height, to first order it has a linear dependence on height. Pressure will decrease linearly both inside and outside the structure, but if the temperatures are different the slopes will be different. Hence, the pressure drop (otherwise called the surface pressure,) will also be linear in height with a slope given by the difference in the interior and exterior slopes. Accordingly, there will be some height at which the internal pressure is equal to the external pressure; this height is the neutral pressure level. If

the temperature inside is greater than out, then below the neutral level the outside pressure is larger than the inside pressure while above it the opposite occurs. The dependence of the stack effect pressures on height is fixed by the temperatures. But the actual pressure difference depends on the height of the neutral pressure level. Referring to Eqs 3 and 5 the infiltration is a function of the neutral pressure level because this level determines which air flow will be larger, Q^+ or Q^-.

WIND EFFECTS

The above discussion makes no reference to wind effects. In this paper we make a set of simplifying assumptions about the effects of the wind:

1) the wind pressure is uniform on each face and

2) is proportional to the square of the wind velocity;

3) only steady state terms are important.

Thus the wind pressure on a face can be expressed as

$$[\text{wind pressure on face j}] = C_j \; \frac{1}{2}\rho v^2 \qquad\qquad 7$$

where the shielding coefficient, C_j, is a function of wind angle for each face.

In general the shielding coefficients will be functions of both wind direction and terrain orientation, but we assume that the shielding is approximately the same everywhere around the house. Then the shielding coefficients will be functions of incident wind direction only. We take j=1 to be the face on which the wind is incident with j increasing clockwise as seen from above. Since we have defined the subscript in terms of the wind direction, we can use as a working hypothesis the assumption that the C_j's are constants. Values of C_j with particular emphasis on shielding effects are reported in studies of Bailey and Vincent (4), Eaton and Mayne (5), Mattingly and Peters (6), and Buckley, Harrje, Knowlton and Heisler (7).

Since we are making the simple assumptions that the shielding coefficients are constant everywhere on a structure face, and that the stack effect is a linear function of height alone, the surface pressure, (as derived in Appendix A), is a linear function of height.

$$\Delta P(\beta) = P_s (\beta_j^0 - \beta) \qquad\qquad 8$$

where

$$P_s = \rho g H \frac{\Delta T}{T_i}$$

$$\beta = \frac{h}{H}$$

$$\beta_j^0 = \beta^0 + (C_j - c^0)\sigma$$

$$\sigma = \frac{\frac{1}{2}\rho v^2}{P_s}$$

β^0 is the height of the neutral level in the absence of wind while β_j^0 is the height of the neutral level for a particular face in the presence of wind. For example, when the wind blows directly on a given face the neutral level will be shifted upward. P_s, the stack pressure, is the difference between the surface pressure at the top of the structure and the bottom of the structure. c^0 is a shielding-like term that is due to the leakage of the wind through the walls. As air moves through a wall the internal pressure can shift to compensate. For example, if one side were very leaky and the structure tight otherwise then c^0 would equal the C_j corresponding to that side, since in steady state the pressure would rise to counteract the wind effect.

Eq 7 can be used to find the surface averaged differential pressure on each face. In the case of the floor or ceiling this results from Eq 7 at the appropriate height; for the vertical faces it is necessary to integrate Eq 7 over the height of the face. These average surface pressures are displayed in Table 1 for all possible values of the neutral level. Fig 1 is a graphical representation of these pressures. Once ΔP_j^+ and ΔP_j^- have been found they can be inserted into Eq 3 to calculate the infiltration.

EXPERIMENTAL TECHNIQUES

In order to calculate infiltration we must know the average surface pressures on each face. These, in turn, can be calculated by knowing the stack pressure, P_s, and the neutral level for that face, β_j^0. The stack pressure can be calculated from the height of the structure and the indoor/outdoor temperatures. The neutral level is a much harder quantity to determine.

There are two approaches to determine the neutral level: either measure the quantity continuously or measure c^0, C_j and β^0 once, assume they remain constant, and predict β_j from these and the wind strength. In either case some direct measurement of a neutral level is necessary. The most direct method of measuring the neutral level is to pierce the envelope with a differential pressure transducer. The pressure, $\Delta P(\beta)$, can then be used to find the neutral level by a straightforward application of Eq 8.

$$\beta_j^0 = \beta + \frac{\Delta P(\beta)}{P_s} \ . \qquad\qquad 9$$

However, except for specially built test structures it is not practical to make penetrations in the envelope. Some method that does not require a penetration in the envelope must be used instead.

Since stack effect pressures are affected by density differences care must be taken not to allow density difference to occur in any test lines used for making pressure measurements. For example, in Ref (3) a line was strung from a tap on the exterior surface, around the structure to an existing opening (e.g. outside air intakes or dryer vent) and into the house to the pressure transducer. Because there was a thermal gradient in the line as it went through the shell, the density of air within the line changed and made this measurement technique insensitive to stack pressures.

A better method would be to measure the interior and exterior pressure against the same fixed reference and subtract to find the difference, but it is impossible to use a reference that is fixed to 1 ppm of an atmosphere. The need for a fixed reference could be eliminated provided that both references were the same; this would allow differential measurements in an acceptable range.

In our measurements we use a helium filled line connecting the interior of the house with the exterior to transmit the pressure information. Helium was chosen because it is the lightest gas available; having a density only $4/29$ that of air, it is relatively unaffected by temperature changes. We use two sensors, one inside and one outside, connected to each other by the helium filled line. If an infinitely elastic membrane were available to hold the helium in the line, only one sensor would be necessary, but the only practical method of containing the helium without losing the pressure information is with a second sensor. Fig 2 is a diagram of a typical experimental set-up. In Appendix B we calculate the actual pressure difference from the two measured differential pressures.

MEASUREMENTS

Using the technique from the previous section it is possible to continuously measure the neutral levels for a structure and from this predict the infiltration. The technique for measuring the neutral level is rather cumbersome for long term testing; it would be better if an easier set of parameters could be measured on a long term basis. Consider the expression for the neutral level:

$$\beta_j^0 = \beta^0 + (C_j - C^0)\sigma \ .$$

$$10$$

The only dynamic parameter is the wind strength, σ. It can easily be measured by a weather tower, which need not be located exactly on site. C_j is, presumably, a continuous function of direction. β^0 is only a function of structural configuration; that is, only the leakiness of the structure will affect the height of the neutral level in no wind. C^0 is both a function of configuration and angle of wind incidence.

In principle, by measuring the neutral levels for a short period of time it is possible to extract the quantities β^0, C^0, C_j by linear regression. Once that is done for all the various configurations normally encountered it is possible to calculate the neutral levels knowing only the weather parameters. Once the secondary quantities, β^0, C^0, C_j have been determined, only the primary quantities, T_i, T_e and v, are necessary to predict the infiltration.

SPECIAL CASES AND EXAMPLES

It is difficult, in general, to determine the values of β^0 and c^0; however with some plausible assumptions they can be estimated. The C_j's can be approximated from the tables in Refs. (4) — (7). In this section we shall assume that we are dealing with a perfect cube as the structure and that the leakage function is the same on all sides:

$$Q^{\pm} = \frac{1}{6} L \sum_{j=0}^{5} \Delta P_j^{\pm} \quad .$$

11.1

If there are no vents in the structure then Q^+ must equal Q^- and β^0 must equal $\frac{1}{2}$ by symmetry. Thus we see that the common assumption that the neutral level is half the building height is a consequence of assuming no vents and a vertical symmetry in the leakage pattern. Using Table 1 with $\beta_j^0 = \frac{1}{2}$ we can find the infiltration.

$$Q^+ = Q^- = \frac{1}{16} L P_s$$

11.2

Some typical values for tight mid-west houses are listed below:

T_i $= 21^{\circ}C$ $(70^{\circ}F)$

T_e $= -9^{\circ}C$ $(25^{\circ}F)$

H $= 2.5m$ (8.2 ft.)

L $= 50 \text{ m}^3/\text{hr-Pa}$ $7 \text{ ft}^3/\text{hr-in.}$ (H_2O)

which yield using this model,

P_s $= 3 \text{ Pa}$ $(.012 \text{ in.} (H_2O))$

Q $= 9.5 \text{ m}^3/\text{hr}$ (5.6 cfm) .

However, in any real situation there will probably be vents or some non-symmetrical leakage which would move the neutral level either up or down; this would cause a rapid increase in the infiltration (cf. Fig 3). It is possible that a tall chimney could move the neutral level above the ceiling. Fig. 3 is a graph of the predicted infiltration vs the height of the neutral level. The deviation from Eq 11 is rapid as the value of β^0 changes away from $\frac{1}{2}$.
A quantitative measure of the wind strength is given by σ:

$\sigma \ll 1 \quad \rightarrow \quad$ light wind

$\sigma \approx 1 \quad \rightarrow \quad$ intermediate

$\sigma \gg 1 \quad \rightarrow \quad$ heavy wind .

Evaluating the intermediate case of $\sigma = 1$ will give an estimate of the point at which stack and wind effects are equally important.

$$\frac{\Delta T}{v^2} = 6 \ [\frac{C^0}{(m/s)^2-story}]$$

$$= 3 \ [\frac{F^0}{mph^2-story}]$$

Thus for a $25^0C(45^0F)$ temperature difference the wind must be much faster than 2m/s(4mph) on a single story structure to be considered heavy. Of course this is modified by the shielding around the structure.

When the wind is either very weak or very strong the analysis can be extended.

Light Wind Case

When there is a light wind there will be a shift in each of the β_j^0's away from β^0 that is small compared to one. Since the behavior of the envelope changes when one of the neutral levels crosses the floor or ceiling boundary, it is neither sufficient nor necessary to assume that the shift is small; β^0 may be close to 0 or 1. Rather we assume

$$0 \le \beta_j^0 \le 1 \tag{12}$$

for all j.

From Table 1 we can get the average surface pressure in the light wind case. Remembering that j=0 is for the floor, j=5 is for the ceiling, and j=1,2,3,4 is for the walls,

$$Q^+ = \frac{1}{6} \ LP_s (\beta_5^0 + \sum_{j=1}^{4} (\beta_j^0)^2) \tag{13.1}$$

$$Q^- = \frac{1}{6} \ LP_s (1 - \beta_0^0 + \frac{1}{2} \sum_{J=1}^{4} (1 - \beta_j^0)^2) \ . \tag{13.2}$$

If we assume, as above, that there are no vents then 0^+ must equal Q^- and β^0 must be $\frac{1}{2}$. Inserting this into either of the above equations we find

$$C^0 = \sum_{j=0}^{5} C_j \ . \tag{14}$$

This result, like the result $\beta^0 = \frac{1}{2}$, applies only in the limit of ventless, symmetric leakage. Just as β^0 is pulled toward the height at which a leak occurs, C^0 approaches the C_j of the leakiest face.

Notice that to this point there has been some quadratic dependence of the infiltration on β_j^0. This indicates that the temperature effects (P_s, β^0) and the wind effects (σ, C^0) are not separable. That is, there are cross terms of the form $P_s \beta^0 \sigma C$ which mix the effects of wind and temperature. This implies that it will not be possible to separate the wind and stack effects by linear regression techniques.

Heavy Wind Case

For the light wind case we assume all neutral levels to be between the floor and the ceiling. A heavy wind is just the opposite:

$$\beta_j^0 < 0 \quad \text{or} \quad \beta_j^0 > 1 \qquad\qquad 15$$

for all j.

Referring to Table 1 we see that in a heavy wind there are no terms in either ΔP^+ or ΔP^- for any face that have a quadratic dependence on β_j^0. Therefore there are no terms that mix the stack effect with the wind effect. From this we can conclude that in a high wind the infiltration will be of the form

$$Q = b \, LP_s + c \, L \, \tfrac{1}{2}\rho v^2 \qquad\qquad 16$$

The two overall constants, b and c, relate infiltration to stack effect and wind pressure; b only depends on the configuration and c is approximated by the weighted sum of the shielding coefficients.

The intermediate case, where some of the neutral levels are between floor and ceiling while others are not, cannot be extended in general. But by application of the results shown in Table 1, the infiltration can be calculated on an individual basis.

DISCUSSION

In this paper we have extended our previous models relating infiltration to average surface pressures on the envelope(2,3). Those model assume that flow through the building envelope is linearly proportional to the indoor-outdoor pressure difference. Detailed measurements(8) show that for crack widths up to 1 mm this is a good assumption. In (2) we experimentally examined the assumption that the flow is proportional to the average positive or negative surface pressure when this pressure was caused by the wind and when all major vents were sealed. Agreement between predicted and measured infiltration rates was excellent. The work was extended in (3) to include houses in their normal operating conditions (several with open vents) some of which were located in the northern part of the United states (where stack effects can be important). In this case the agreement between measured and predicted infiltration values was not as good. The ratio was~ 1.35. This was encouraging but indicated the need for a refined model.

This paper describes a general model which will accommodate wind pressures, stack effect and ventilation openings provided the vents are either all above or all below their respective neutral pressure level. In this case the flows through the vents are all into or out of the building. The predicted infiltration is then the larger of the calculated infiltration or exfiltration.

The main innovation in this model is expressing the infiltration as a function of the shell leakage and of the neutral heights of each face, β_j^0, that can in turn be determined by the weather parameters and by the leak distribution or measured directly. The model further introduces shielding coefficients, C_j, which are semi-empirical constants that scale the wind velocities and directions measured at a weather tower or station to produce the resulting surface pressures.

The procedures in this paper are complex; we feel that this complexity is necessary in order to define the areas in which simplifications to the model may be made. For example, our results point out the difficulty in attempting to find linear fits of infiltration rates with ΔT and v or ΔT and v^2. Our results indicate that this procedure will not succeed except in strong wind situations.

This work is continuing in several directions. Ultimately we seek to define a simple set of measurements which can be used to characterize the natural ventilation of a house. Before that goal can be reached, however, several extensions of the work described herein must be made. We must go beyond the simple manner in which wind was treated in this paper. Measurements must be made of surface pressure distributions in simplified geometries when wind direction and speed, temperatures and leakage openings can be varied. Infiltration must be measured when the ventilation openings are located both above and below the neutral pressure level; this condition must also be included in the predictive model. The effects of pressure fluctuations should also be examined and included in the model. When these results are obtained we shall be much closer to a correlation between pressurization and natural infiltration.

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SYMBOL TABLE

A Total surface area

A_j Surface area of face j

C_j Wind shielding coefficient for face j

c^0 Wind response factor

e as a subscript refers to exterior of structure

g Acceleration of gravity

h Height

H Structure height

i as a subscript refers to interior of structure

j as a subscript refers to an exterior face of the structure.
   (j=0 for the floor, j=1,2,3,4 for the walls with j=1 being the wall with the wind incident on it (increasing clockwise), and j=5 for the ceiling)

l   Specific leakage [velocity/pressure]

L   House leakage [flow/pressure]

M   Molar mass of air

M_1 Molar mass of light gas

P   Pressure

P_a Atmospheric Pressure

P^0 Internal reference pressure

P_s Stack pressure

Q   Total Infiltration

Q^+ Infiltration due to positive surface pressures

Q^- Exfiltration due to negative surface pressures

R   Ideal gas constant

T   Absolute temperature

v   Wind speed

β Height divided by structure height

β^0 Normalized neutral pressure level

ΔP Differential pressure

ΔP^+ Area averaged positive surface pressure

ΔP^- Area averaged negative surface pressure

ΔQ Difference between infiltration and exfiltration (vent flow)

ΔT Absolute inside-outside temperature difference

ρ Density of outside air

σ Wind strength

APPENDIX A

In this appendix we will derive the average surface pressures induced on a structure by a combination of stack effect and wind. The stack effect is caused by a difference in density between inside and outside air which is due to a temperature difference between inside and out. Even with large temperature differences, the pressure differences induced are very small compared to atmospheric pressure (≈ 10 Pa vs $\approx 10^5$ Pa) so some ambiguity exists for defining absolute references. We take the exterior of the structure in no wind at ground level to be our reference; that is, atmospheric pressure is the pressure at that point; our measurement of height begins at that point as well.

If we assume the atmosphere near the surface is isothermal then the pressure can be calculated as a function of height.

$$P_e(h) = P_a e^{-\frac{Mgh}{RT_e}}$$

A1

Inside a structure the atmosphere can be treated as isothermal also, but with two differences:

1) The temperature will be different and

2) the pressure at zero height could be different

$$P_i(h) = P^0 e^{-\frac{Mgh}{RT_i}}$$

A2

where P^0 is as yet undetermined.

Using the ideal gas law for outside air,

$$MP_a = \rho RT_e$$

A3

and normalizing the height to the height of the structure ,

$$\beta = \frac{h}{H}$$

A4

Eq. A1 and A2 become,

$$P_e(\beta) = P_a e^{-\frac{\rho g H}{P_a}\beta}$$

A5

$$P_i(\beta) = P^0 e^{-\frac{pgH}{P_a}\frac{T_e}{T_i}\beta} \qquad\qquad A6$$

Since pgH is typically about 30 Pa the exponents in the above equations are very small and thus the exponentials can be expanded to first order.

$$P_e(\beta) = P_a - pgH\,\beta \qquad\qquad A7$$

$$P_i(\beta) = P^0 - pgH\,\beta\,\frac{P^0}{P_a}\frac{T_e}{T_i} \qquad\qquad A8$$

Since P^0 differs from P_a by less than one part per thousand, no significant error is introduced by replacing it in the last term above; that term is already less than one-thousandth of an atmosphere.

$$P_i(\beta) = P^0 - pgH\,\beta\,\frac{T_e}{T_i} \qquad\qquad A9$$

The neutral level, β^0, is the level at which the interior pressure and exterior pressure are equal.

$$P_i(\beta^0) = P_e(\beta^0) \qquad\qquad A10.1$$

$$P_a - pgH\,\beta^0 = P^0 - pgH\,\beta^0\,\frac{T_e}{T_i} \qquad\qquad A10.2$$

solving for P^0,

$$P^0 = P_a - pgH\,\beta^0\,\frac{\Delta T}{T_i} \qquad\qquad A11$$

The pressure drop across a vertical face can be calculated in terms of the neutral level, β^0

$$\Delta P(\beta) = P_e(\beta) - P_i(\beta) \qquad\qquad A12.1$$

$$\Delta P(\beta) = pgH\,\frac{\Delta T}{T_i}\,(\beta^0 - \beta) \qquad\qquad A12.2$$

We can now define the stack pressure.

$$P_s = \rho g H \frac{\Delta T}{T_i} \qquad\qquad A13$$

Rewriting some of the above equations in these terms,

$$P_e(\beta) = P_a - P_s \frac{T_i}{\Delta T} \beta \qquad\qquad A14.1$$

$$P_i(\beta) = P_a - P_s \beta^0 - P_s \beta \frac{T_e}{\Delta T} \qquad\qquad A14.2$$

$$\Delta P(\beta) = P_s (\beta^0 - \beta) \qquad\qquad A14.3$$

To include the effects of the wind Eq A14 must be expanded. In our simple model the wind affects the exterior of each face; each face may have a different wind pressure on it, but every part of a given face is affected the same way. Thus, instead of having only one exterior pressure function, we have a different one for each face of the building. Accordingly, we must subscript the exterior pressure to denote the difference.

$$P_e \rightarrow P_j = P_e + C_j \; \tfrac{1}{2}\rho v^2 \qquad\qquad A15$$

where C_j is the shielding coefficient for that face.

While the interior pressure is not directly affected by the wind, it is affected by the average exterior pressure on it. There may be a shift in the interior pressure induced by a change in the exterior pressure profile.

$$P_i \rightarrow P_i + C^0 \; \tfrac{1}{2}\rho v^2 \qquad\qquad A16$$

Introducing, as a dimensionless parameter, the wind strength,

$$\sigma = \frac{\tfrac{1}{2}\rho v^2}{P_s} \qquad\qquad A17.1$$

$$= \frac{v^2}{2gH} \frac{T_i}{\Delta T} \qquad\qquad A17.2$$

we can rewrite Eq A14.

$$P_j(\beta) = P_a - P_s \frac{T_i}{\Delta T}\beta + C_j P_s \sigma \qquad\qquad\text{A18.1}$$

$$P_i(\beta) = P_a - P_s \beta^0 - P_s \frac{T_e}{\Delta T}\beta + c^0 P_s \sigma \qquad\qquad\text{A18.2}$$

$$\Delta P_j(\beta) = P_s(\beta^0 - \beta + \sigma(C_j - c^0)) \qquad\qquad\text{A18.3}$$

Hence there is a height for every face at which the inside and outside pressures are equal. (This height may not actually be between the floor and the ceiling, but would be if the structure were tall enough.) The height may well be different for every face.

$$P_j(\beta_j^0) = P_i(\beta_j^0) \qquad\qquad\text{A19.1}$$

$$\beta_j^0 = \beta^0 + \sigma(C_j - c^0) \qquad\qquad\text{A19.2}$$

Thus the pressure drop across a face is a deceptively simple looking function of height:

$$\Delta P_j(\beta) = P_s(\beta_j^0 - \beta) \qquad\qquad\text{A20}$$

In order to use these surface pressures in the infiltration models they must be averaged over each face for their positive and negative parts. For horizontal faces like the floor and the ceiling this is simple, since Eq A20 meed only be evaluated at the appropriate height. However, since the pressure drop is a function of height, the pressure averages involve an integral over the vertical component of each face. These averages will be functions of only β_j^0.

The problem can be divided into three cases for calculation:

1): $\qquad \beta_j^0 < 0$ (neutral level below floor)

$$\Delta P_j^+ = 0$$

$$\Delta P_j^- = P_s \int_0^1 (\beta - \beta_j^0)\,d\beta$$

$$= \tfrac{1}{2} P_s (1 - 2\beta_j^0)$$

2): $0 < \beta_j^0 < 1$ (neutral level between floor and ceiling)

$$\Delta P_j^+ = P_s \int_0^{\beta_j^0} (\beta_j^0 - \beta)d\beta$$

$$= \tfrac{1}{2} P_s (\beta_j^0)^2$$

$$\Delta P_j^- = P_s \int_{\beta_j^0}^1 (\beta - \beta_j^0)d\beta$$

$$= \tfrac{1}{2} P_s (1 - \beta_j^0)^2$$

3): $1 < \beta^{0_j}$ (neutral level above ceiling)

$$\Delta P_j^+ = P_s \int_0^1 (\beta_j^0 - \beta)d\beta$$

$$= \tfrac{1}{2} P_s (2\beta_j^0 - 1)$$

$$\Delta P_j^- = 0$$

The results of the above calculations are compiled in Table 1.

In this appendix we will derive the expression for the actual pressure difference between any point on the inside of the structure and any point on the outside of the structure, using the measurement scheme proposed in the text. There are two differential pressure sensors, one on the inside of the structure and one on the outside of the structure; their reference ends are connected together by a flexible line filled with a low molecular weight gas (helium). (cf. Fig 2).

To begin, we start with Eq A18 adding subscripts to denote inside and outside height variables.

$$P_j(\beta_j) = P_a + C_j P_s \sigma - P_s \frac{T_i}{\Delta T} \beta_j \qquad \text{B1.1}$$

$$P_i(\beta_i) = P_a - P_s \beta^0 + C^0 P_s \sigma - P_s \frac{T_e}{\Delta T} \beta_i \qquad \text{B1.2}$$

$$\Delta P_j(\beta_i, \beta_j) = P_s (\beta_j^0 + \frac{T_e}{\Delta T} \beta_i - \frac{T_i}{\Delta T} \beta_j) \qquad \text{B1.3}$$

$$\Delta P_j(\beta_i, \beta_j) = P_s (\beta_j^0 + \frac{T_i}{\Delta T}(\beta_i - \beta_j) - \beta_i) \qquad \text{B1.4}$$

Solving for β_j^0,

$$\beta_j^0 = \beta_i + \frac{\Delta P_j(\beta_i, \beta_j)}{P_s} + \frac{T_i}{\Delta T}(\beta_j - \beta_i) \qquad \text{B2}$$

a measurement of $\Delta P_j(\beta_i, \beta_j)$ along with the other quantities will determine β_j^0.

Consider the set-up shown in Fig. 2 where the line connecting the two sensors is filled with a light gas of mass M_1. The measured values from the two instruments can be written in the terms shown in that figure.

$$\Delta P^1 = P_i - P_i^0 \qquad \text{B3.1}$$

$$\Delta P^2 = P_j - P_j^0 \qquad \text{B3.2}$$

The equation of hydrostatic equilibrium can be used to relate the pressures inside the line.

$$P_j^0 = P_i^0 + \rho_1 g H \int_{\beta_i}^{\beta_j} \frac{T_e}{T} \cos\theta \, d\beta \qquad\qquad B4$$

where ρ_1 is the density of the light gas

T is the Temperature at any point in the line

$\cos\theta$ is the angle between vertical and the line path

In general the above path integral will be very complicated. In order to simplify it we will assume (for the moment) that all of the temperature change occurs during a horizontal section at height β'.

Then the equations become,

$$P_j^0 = P_i^0 + \rho_1 g H \frac{T_e}{T_i} \int_{\beta_i}^{\beta'} d\beta + \rho_1 g H \int_{\beta'}^{\beta_j} d\beta \qquad\qquad B5.1$$

$$= P_i^0 + \rho_1 g H [(\beta' - \beta_i) - \frac{\Delta T}{T_i}(\beta' - \beta_i) + (\beta_j - \beta')] \qquad\qquad B5.2$$

$$= P_i^0 + \rho_1 g H (\beta_j - \beta_i) - \rho_1 g H \frac{\Delta T}{T_i}(\beta' - \beta_i) \qquad\qquad B5.3$$

The last term is smaller than the previous term by a factor of $\frac{\Delta T}{T_i}$, and the previous term was smaller than any other by a factor of $\frac{\rho_1}{\rho}$. For this reason we can justify neglecting the last term as long as the mass of the light gas is small compared to the mass of air. If the temperature change does not occur over a horizontal section then only the last term is affected. Thus if a light gas is used the last term can be neglected for any reasonable line path.

$$\Delta P_j (\beta_i, \beta_j) = \Delta P^2 - \Delta P^1 + \rho_1 g H (\beta_j - \beta_i) \qquad\qquad B6$$

Combining this with Eq B2,

$$\beta_j^0 = \beta_i + \frac{\Delta P^2 - \Delta P^1 + \rho_1 g H (\beta_j - \beta_i)}{P_s} + \frac{T_i}{\Delta T}(\beta_j - \beta_i) \qquad\qquad B7$$

and using the definitions of the stack pressure and gas density,

$$\beta_j^0 = \beta_i + \frac{\Delta P^2 - \Delta P^1}{P_s} + (1 + \frac{M_1}{M}) \frac{T_i}{\Delta T}(\beta_j - \beta_i) \qquad\qquad B8$$

which only differs from Eq B2 by a correction factor for the light gas.

Eq B8 relates the measured quantities $(\beta_i, \beta_j, \Delta P^1, \Delta P^2)$ to the desired quantity, β_j^0. Note also that Eq B6 can be used to measure the absolute pressure difference between two points at the same temperature.

TABLE 1

Table of the positive average differential surface pressures as a function of the height of the neutral level for that face.

$\dfrac{\Delta P_i^+}{P_s}$	$\beta_j^0 < 0$	$0 \le \beta_j^0 \le 1$	$1 < \beta_j^0$
ceiling	0	0	$\beta_j^0 - 1$
walls	0	$\frac{1}{2}(\beta_j^0)^2$	$\beta_j^0 - \frac{1}{2}$
floor	0	β_j^0	β_j^0

TABLE 2

Table of the negative average differential surface pressures as a function of the height of the neutral level for that face.

$\dfrac{\Delta P_i^-}{P_s}$	$\beta_j^0 < 0$	$0 \le \beta_j^0 \le 1$	$1 < \beta_j^0$
ceiling	$1 - \beta_j^0$	$1 - \beta_j^0$	0
walls	$\frac{1}{2} - \beta_j^0$	$\frac{1}{2}(1 - \beta_j^0)^2$	0
floor	$-\beta_j^0$	0	0

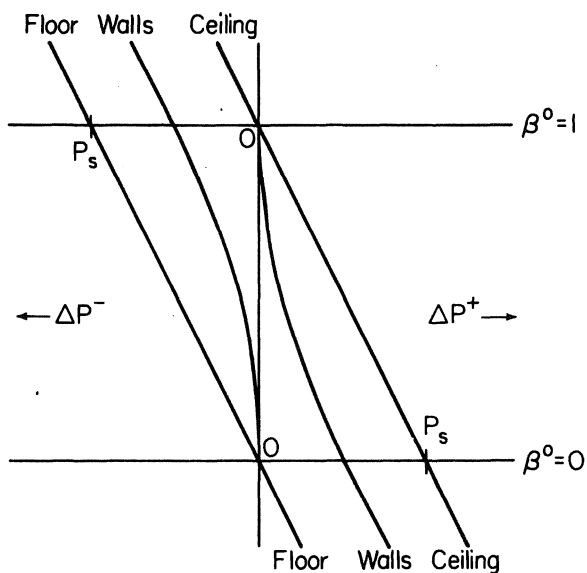NOTE: If $\Delta T < 0$ then ΔP^+ and ΔP^- interchange values from the ones given above.

Fig. 1 Average surface pressures (horizontal scale) vs height of neutral pressure level (vertical scale). β is the height of the neutral pressure level divided by the structure height ΔP^+ is the positive differential surface pressure averaged over the surface. ΔP^- is the negative differential surface pressure averaged over the surface.
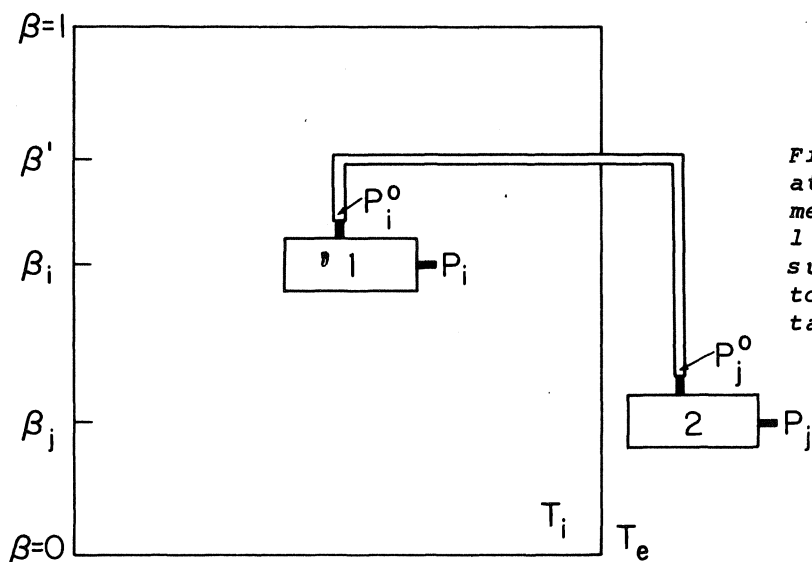

Fig. 2 Experimental configuration for neutral pressure level measurement. The boxes labeled 1 and 2 are differential pressure sensors which are connected together by a reference line containing helium gas.
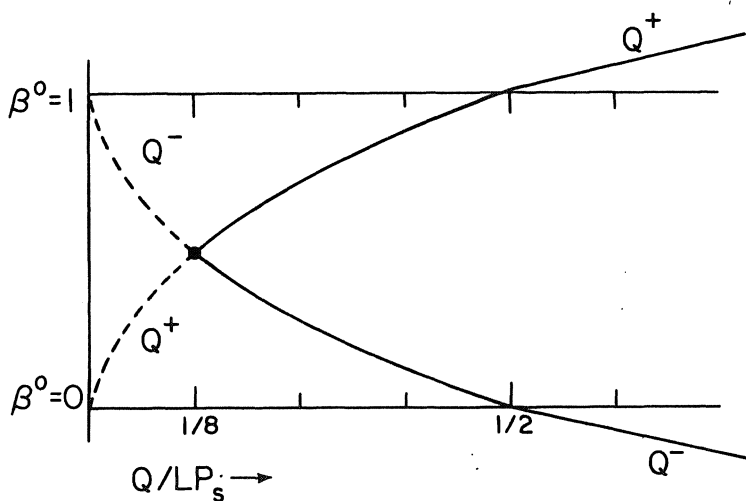

Fig. 3 Predicted infiltration rate vs neutral level for the no wind case. At a particular pressure level, the solid curve (the larger of Q^+ and Q^-) predicts the infiltration rate.

When the neutral pressure level is at the midpoint of the structure, ($\beta = 1/2$), the two flows Q^+ and Q^- are equal. The difference between Q^+ and Q^- for other values of β represents the net flow through vents or other large openings.