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Critical Significance of Attics and Basements in the Energy Balance of Twin Rivers Townhouses

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Approximately 35% of winter energy loss in Twin Rivers townhouses is associated with the attic, despite the presence of 9 cm of fiberglass insulation. Unexpected heat transfer mechanisms bypass the attic insulation and strongly couple attic, basement and house. As a result, a three-zone model is required for static heat load calculations and the prediction of retrofit savings.

Magnitudes of the unexpected heat transfer rates can be inferred from attic and basement temperatures and knowledge of furnace inefficiencies. The model predicts the benefits to be gained by various retrofit strategies. Effectiveness of retrofits may be considerably enhanced by blocking heat transfer bypass paths to the attic.

INTRODUCTION

The thermal performance of attics and basements plays an important role in determining the use of energy in residential structures. The heat transfer from living space to attic frequently represents a significant energy loss. Radiative heat from the furnace in an unused basement leads to further waste.

Several ways have been suggested for reducing energy use by altering the characteristics of the attic: attic insulation, reduction of attic ventilation by closing vents, and use of attic fans. Recommendation of attic insulation, at least, has become an accepted part of government policy. This policy has largely been based on attic thermal models which, in predicting savings to be obtained by adding attic insulation [1 - 3], only consider conduction through the attic floor and roof (and sometimes include ventilation). However, when these standard models are applied to attics in Twin Rivers townhouses, predicted attic temperatures in winter are consistently much lower than the actual measured values [4]. The high measured attic temperatures imply that the attic heat losses are much higher than predicted by standard models. In fact, it is our conclusion that the heat loss through an average Twin Rivers attic is about five times as much as the theoretically predicted value and accounts for 35% of the energy leaving the house in winter — even though the attic usually contains 9 cm (R-11) of fiberglass insulation.

Two major reasons for the ineffectiveness of the attic insulation are: (a) air flow into the attic from other parts of the house, and (b) heat transfer into the attic by way of the party walls between adjacent houses*. These heat transfer mechanisms bypass the installed fiberglass attic insulation. If these bypass mechanisms had been eliminated at the time of construction, considerable savings would have resulted. Experiments have shown that similar savings can be expected from retrofitting existing Twin Rivers attics.

The Twin Rivers attic discrepancy should serve as a warning that better understanding of the thermal performance of attics is essential before the results of attic retrofits can be accurately predicted nationwide. If bypass mechanisms are a common attic phenomenon, then improving the thermal performance of attics is more complicated than currently

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*While party walls are only found in attached housing, similar large air flow losses through the attic are probably also common in pre-1940 U.S. detached houses where "balloon-frame" construction was the rule.

presumed. On the other hand, the Twin Rivers discrepancy also indicates that on a nationwide basis the potential energy savings to be obtained from the attic may be larger than previously estimated.

Our analysis of the bypass mechanisms at Twin Rivers, presented in detail in another paper [5], has shown that the attic is strongly coupled both to living space and to basement. As a result, a three-zone model is required for even a static heat load calculation. The attic and basement cannot be treated separately in predicting total energy use.

In the first section of this paper we review the discrepancies found in Twin Rivers attics and our resolution of the problem. In the second section we deal with attic losses in the context of total energy leaving the house. The three-zone model for energy flow is developed, and a procedure is outlined for obtaining net furnace efficiency. We also discuss the theoretical justification for a simple diagnostic test for rating attic performance — namely, the two-resistance model test.

THE WARM-ATTIC DISCREPANCY

Visual examination of Twin Rivers attics suggests some causes for the warm attics. The furnace flue passes through the attic and is surrounded by an open shaft to isolate the flue from other building materials*. Warm air enters the attic not only through the shaft but also through cracks between the attic floor and the party walls that separate adjacent attics.

As part of an experiment to measure savings in gas use in a number of Twin Rivers houses [6], the shaft surrounding the furnace flue was sealed (D retrofit), additional insulation was added to the attic floor and cracks between house and party wall were sealed in the attic (A retrofit)**.

These improvements led to a cooling of the attic, but after retrofits the attic was still warmer than expected. The remaining discrepancy was not eliminated until batts of fiberglass insulation were glued to the attic party walls, isolating the party wall from the attic.

The attic thermal discrepancies may be expressed in terms of a fit to an approximate physical model which is equivalent to a static heat load calculation with variable parameters. As we shall see, the discrepancy is so great that it dwarfs all the approximations in the model. This model assumes that: (a) the house interior, the attic and the outside may each be represented by a single temperature (T_H, T_A, and T_O respectively). We include basement and neighboring houses as part of our definition of house interior†; (b) the heat flux rate between adjacent regions is proportional to the temperature difference between them; these proportionality constants are referred to as effective *conductances* and are identified by W_{HA} and W_{AO} for house-to-attic and attic-to-outside heat flow, respectively; (c) there are no internal "sources" of energy in the attic; (d) the thermal storage effects of the attic are negligible during measurement periods; (e) the effects of solar radiation on the roof exterior are negligible. The physical content of the model may then be stated as an equality of two heat flows:

$$W_{HA}(T_H - T_A) = W_{AO}(T_A - T_O) \tag{1}$$

The instantaneous heat transfer rate from house to attic must equal that from attic to outside. It is convenient to rewrite this equation and to define a temperature ratio, λ:

$$\lambda = \frac{W_{AO}}{W_{HA}} = \frac{T_H - T_A}{T_A - T_O} \tag{2}$$

If both the conductances, W_{AO} and W_{HA}, are constants over time, the attic temperature will keep the same relative position between the inside and outside temperatures. We call either eqn. (1) or (2) the "two-resistance model."

In traditional attic energy loss calculations, W_{HA} is the linearized "UA" value for heat

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*Building codes often require this for fire safety reasons. We have verified that there are ways to block the furnace shaft with fire-resistant materials.

**Details of the "ABCD" retrofits referred to in this paper can be found in D. Harrje's article in this issue [11].

†Twin Rivers basements typically have roughly the same temperature as the living space. Similarly, neighbor temperatures can be considered the same, to first approximation. In any case, as shown in the Appendix, an effective two-resistance model should hold even if the basement and neighbors are at different temperatures.

transfer through the attic floor and W_{AO} includes heat transfer through the roof and attic ventilation. For a typical Twin Rivers attic before retrofits, using standard handbook thermal properties and standard attic ventilation rate, we find $W_{AO} = 290$ W/°C and $W_{HA} = 33$ W/°C; hence the value of the ratio λ in eqn. (2) is predicted to be about 9*.

In a statistical analysis of the temperature ratio [9], the temperature data were restricted to six hour night-time averages from midnight to 6 a.m. to increase the validity of assumptions (d) and (e) of the model†. The analysis showed that the average value of λ for the houses in the sample, measured over a period of 83 days, was 1.0, and not the predicted value of 9 [9].

The first question to answer about the factor of 9 discrepancy is whether it could be due to well-known inadequacies in the model. A detailed analysis [10] showed that effects included in complex computer programs such as NBSLD, but left out of the elementary static calculation (such as storage and radiation heat transfer), could not possibly explain such a large discrepancy. Furthermore, direct and indirect heat flux measurements ruled out the possibility that the discrepancy was the result of inaccurate estimates of the thermal resistances of attic floor or roof [5]. Yet a statistical analysis showed that, late at night, the attic temperature did remain in fixed proportion (with a standard deviation of 1 °C) between the house and outside temperatures, suggesting that a two-resistance model was empirically valid provided that the conductances took on non-handbook values. This suggests that a mechanism for additional heat transfer from house to attic exists characterized by a conductance (W_P) in parallel with the conductance through the attic floor. We decided to use measured attic temperature and the two-resistance model as a diagnostic tool to search for the missing parallel heat path. We modified the attic in various ways and kept checking the temperature ratios statistically to see how closely the measured value approached the predicted one.

Air flow directly into the attic was an obvious candidate for an additional conductance. However, A and D retrofits previously referred to were sufficient to block most air flow into the attic, as shown by tracer gas air infiltration measurements. Thus, if air flow were the only significant bypass mechanism, one would expect agreement between the post-retrofit value of the temperature ratio λ observed in the field and the value calculated from the thermal properties of the materials in the retrofitted attic. This agreement was not found. To be sure, the attic became cooler after retrofits but a large discrepancy still remained: a calculation for retrofitted attics (with additional attic insulation) using handbook thermal properties led to a prediction of $\lambda = 23$. The experimental value for the ratio, averaged over 13 townhouses was much smaller, equal to 2.5 [9]. This discrepancy suggests that additional heat transfer paths exist other than air flow directly into the attic.

We have narrowed this remaining discrepancy to heat transfer within the wall of cinder blocks (the party wall) that divides adjacent townhouses from each other. The cinder blocks are 20 cm thick and the wall extends from the basement to the attic. In the two inhabited floors, each wall is faced with gypsum board, which is separated from the cinder blocks by an air space. In the attic and basement the cinder blocks are generally uncovered.

Cinder blocks are so constructed as to leave large holes in them. In the wall, they are stacked up in such a manner that the holes in the cinder blocks are vertical, although not necessarily well aligned. Heat flow through the wall in the vertical direction takes place both by conduction through the solid portions of the wall and by air movement through the vertically connected holes. Theoretical estimates suggest that the conductive component of heat transfer is small but a significant amount of heat may be added into

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*We use nominal conductivities for the fiberglass and the attic structural materials and make a correction for the geometry of joists [7]. We have confirmed the conductive heat flow by direct measurements at attic floor and roof. For ventilation rates (not measured) we use the value 0.039 m^3/s using the model in Hinrichs and Wolfert [8] and assuming a wind speed of 3 m/s. We refer to the results of these calculations as "handbook values" of the conductances W_{HA} and W_{AO}.

†Storage effects are minimal when temperatures change slowly with time. This occurs in the early morning hours before sunrise.

TABLE 1

Estimated "house"-to-attic conductances for an average townhouse[e] (W/°C)

	pre-retrofit	post ABCD retrofit
attic insulation	33	13
Air flow bypass through furnace shaft and cracks between building and party wall[a]	160[c]	5[d]
party wall bypass conductance[b]	74	80
remaining discrepancy	23	20
TOTAL = W_{HA}	290[f]	116

[a] Includes air flow from basement as well as convection from air spaces which connect to attic.

[b] Includes any contribution from neighbors.

[c] Derived from change in missing conductance following ABCD retrofit.

[d] Result of SF_6 tracer gas measurements in the experimental townhouse (which was made especially tight).

[e] "House" includes basement and neighbors.

[f] Because attic temperatures lie midway between inside and outside temperatures, this number must equal the total roof conductance (measured experimentally to lie within ±20% of 290 W/°C).

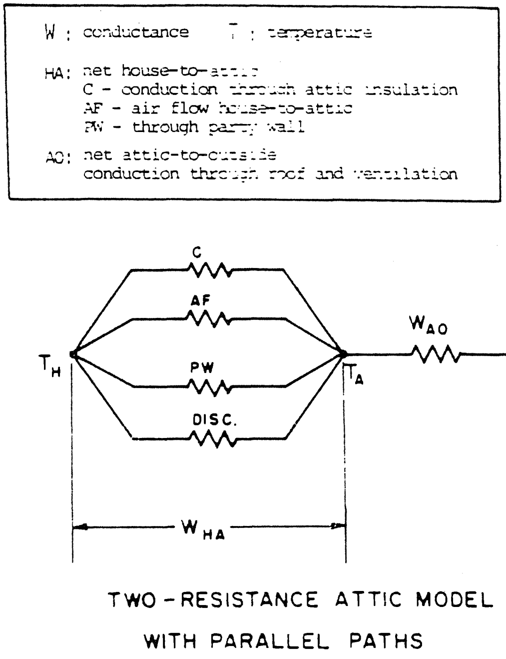

Fig. 1. Two-resistance attic model with parallel paths.

the attic by convection within the cinder block cavities [5].

These theoretical estimates were supplemented by an experiment to measure the extent of attic heating from the party walls [5]. Batts of R-11 fiberglass insulation were cut to shape and attached to both party walls in the attic of the experimental townhouse. The attic had already received the A and D retrofits, *i.e.*, it had R-30 insulation on the floor and all cracks along the party wall, as well as the furnace shaft, were sealed. Following the installation of the insulation on the party walls, the measured value of λ for the first time approached the value predicted by traditional methods. The attic was at last as cold as it was supposed to be.

As a result, we conclude that the party wall thermally bypasses the attic insulation and represents the main source of discrepancy remaining after air flow is blocked.

We can estimate the magnitude of the unexpected conductances (associated with air flow and party wall heat transfer) using another representation of the two-resistance model, shown in Fig. 1. W_{AO} is the conductance due to conduction through the roof and attic ventilation. The conductance between house interior and attic (W_{HA} in the two-resistance model) is made up of a set of parallel conductance paths resulting from conduction through the attic insulation (C), by air flow from house to attic (AF), and through the party wall (PW). For convenience, any remaining discrepancy between the two-resistance model predictions and experimentally measured attic temperatures is expressed in terms of another parallel conductance between house and attic (DISC). These conductances are shown in Table 1.

The principal result indicated in Table 1 is that the attic discrepancy is the result of large heat bypass paths between house and attic (by air flow and through the party wall) before the retrofits. The retrofits reduce the conductances through the attic insulation and by air flow but the party wall bypass conductance remains dominant. This has a major impact on the cost-effectiveness of conservation strategies at Twin Rivers.

The discrepancies which remain in Table 1, cannot be considered serious, given the accuracy of our measurements and the approximate nature of the two-resistance model. We conclude that attic temperatures (and consequently attic heat fluxes) can be predicted reasonably accurately in Twin Rivers houses if the two insulation bypass mechanisms (air flow and party wall) are taken into account. Of course, there are other effects not included in our discussion (all of which we considered in unraveling the discrepancy) which play a smaller role in determining attic temperatures at night, but which may be important at other times or for other types of attics. These effects are: (1) thermal lag introduced by the thermal capacitance of the attic materials, (this has been shown to be negligible for the time period considered experimentally, but would be important at times when the attic temperature is changing rapidly); (2) spatial inhomogeneity of temperature in attics; (3) variations in the wind and the radiation environment of the exterior, (including sun and night sky cooling); (4) details of radiation heat transfer, e.g. from roof-to-wall-to-floor; (5) heat gain in the attic due to the furnace flue, and from other metal penetrations into the attic. (Although this effect is relatively small at Twin Rivers, the heat flux from the furnace flue might be important in making an economically optimal decision about insulation thickness in other attics.)

Some of these effects are already included in modern-day heat-load computer codes. However, even a complex computer code, if it does not include attic bypass mechanisms adequately, is no more useful for attics than an old-fashioned handbook estimate. How best to modify existing codes to take into account attic bypass mechanisms (if supporting measurements are unavailable) may become clear as familiarity is gained with a variety of attic types.

HOUSE ENERGY USE

The fact that the party wall is responsible for a large conductance bypass into the attic, suggests that the basement is probably also thermally coupled to the attic, since the party walls are bare in the basement and capable of absorbing substantial basement heat. If the basement–attic coupling is large, then an energy balance equation for the basement will reveal the same coupling.

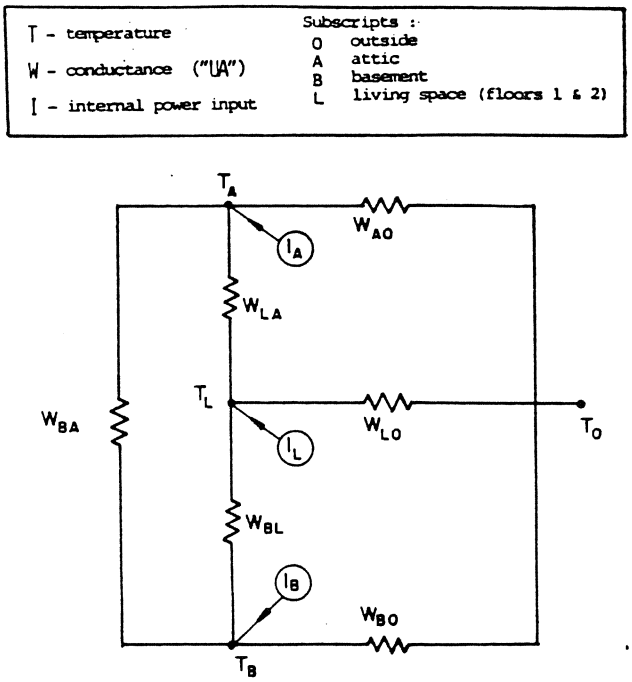

Fig. 2. Three zone model — equivalent circuit.

A simple steady-state heat flow circuit with three zones is shown in Fig. 2. This is an extension of the two-resistance attic model described earlier and assumptions analogous to (a), (b), (d) and (e) for the two-resistance model are made.

This model does not include any thermal capacities, thus neglecting the effects of thermal storage. Storage effects become important if the house warms up above the thermostat setting for part of the day, as happens in mild weather, or if very short time periods are modeled. However, the average power (averaged over periods of a day or more) passing through a thermal circuit is insensitive to storage effects, provided the house temperature remains roughly constant.

In the present model, it is assumed that constant fractions of the furnace combustion power are deposited in the attic, the basement, and exhausted at the top of the flue. The remainder of the furnace power, together with power from appliances, people and sun are released in the living space. Internal heat sources for the living space, basement, and

TABLE 2

Allocation of furnace heat between attic, basement, flue and living area

		pre-retrofit (%)	post-ABCD retrofit (%)	Source
attic	γ_A	4	1	estimate
basement	γ_B	26	13	[7], estimate
flue	γ_C	16	18	[12], estimate
living area	γ_L	54	68	$\gamma_L = 1 - \gamma_A - \gamma_B - \gamma_C$

attic are designated I_L, I_B and I_A, respectively in Fig. 2. They are given by:

$$
\begin{aligned}
I_L &= (1 - \gamma_A - \gamma_B - \gamma_C)I + I_{\text{free}} = \gamma_L I + I_{\text{free}} \\
I_B &= \gamma_B I \qquad (3) \\
I_A &= \gamma_A I
\end{aligned}
$$

where I is the average total combustion power at the specified outside temperature, I_{free} is the average power from appliances, people and sun. γ_A, γ_B, and γ_C are the fractions of I dumped in the attic, the basement and leaving the furnace flue, respectively. The values of γ change after certain retrofits are made. Estimated values of γ before and after the 'ABCD' set of retrofits (described in ref. [11]) are shown in Table 2.

The three-zone model of the house leads to the following coupled equations for energy balance:

(a) basement energy balance:

$$W_{BA}(T_B - T_A) + W_{BL}(T_B - T_L) + W_{BO}(T_B - T_O) = \gamma_B I \qquad (4)$$

(b) attic energy balance:

$$W_{BA}(T_B - T_A) + W_{LA}(T_L - T_A) + \gamma_A I = W_{AO}(T_A - T_O) \qquad (5)$$

(c) living space energy balance:

$$W_{LO}(T_L - T_O) + W_{LA}(T_L - T_A) = \gamma_L I + I_{\text{free}} + W_{BL}(T_B - T_L) \qquad (6)$$

The set of coupled energy balance equations can be solved for the furnace combustion rate necessary to keep the living space temperature, T_L, constant for a given outside temperature, T_O. An expression for I may be written as:

$$I = (W_{\text{net}}(T_L - T_O) - I_{\text{free}})/\epsilon \qquad (7)$$

where W_{net} is an overall conductance of the living space and is determined by the values of the W's*; ϵ is the efficiency of the furnace (relative to a loss-free heat source within the living space) and depends on the W's as well as γ's.

The same set of simultaneous equations determines the attic and basement temperatures. As shown in the Appendix, the equation for the attic temperature retains an approximate two-resistance form — an explanation of why we found attic temperature data behaving so regularly.

Reliable estimates of I_{free} and the γ values are available from earlier work at Twin Rivers. Estimates of W_{BL}, W_{LO}, W_{BO}, and W_{AO} can be obtained from handbook values of materials properties [7] and measured air infiltration data.

The unknown bypass conductances, W_{BA} and W_{LA}, must be determined from measured temperature data, using eqns. (4) and (5). A feature of Twin Rivers townhouses is that, prior to retrofits, their basements are at a temperature close to the living space. As a result, although the solution of eqns. (4) and (5) for W_{BA} and W_{LA} are sensitive to changes in estimates of the remaining W's, the sum of W_{BA} and W_{LA} is relatively insensitive. The sum of W_{BA} and W_{LA} turns out to approximate the total bypass conductance determined from the two-resistance model. To obtain individual values for W_{BA} and W_{LA}, we have used night-time furnace and temperature data from five townhouses — those without modifications to their basements. Since W_{LA} does not

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$$*W_{\text{net}} = W_{LO} + \frac{1}{W_{BA}}\left[\frac{pq - mn}{p + q + m + n}\right] \qquad (8)$$

where
$p = W_{BA}(W_{LB} + W_{LA}) + W_{LA}W_{LB}$
$q = W_{BO}W_{AO} + W_{BA}(W_{AO} + W_{BO})$
$m = W_{LB}W_{AO}$
$n = W_{LA}W_{BO}$

TABLE 3

Conductances in the three-zone model (W/°C). W_{AO} = 290, W_{BL} = 330 (handbook estimates)

	W_{LO}[a]	W_{BO}[a]	W_{BA}	W_{LA}	W_{net}	ϵ
As built	174	53	105	171	343	0.75
Plus ABCD retrofits [11]	174	53	40	80	294	0.79
Plus hypothetical party wall retrofit est.)	174	53	0	16	234	0.79
Super-retrofit [13]	84	32	40	80	190	0.79
Plus hypothetical party wall retrofit (est.)	84	32	0	16	128	0.80

[a] Air infiltation rates were included using typical measured air exchange rates.

[b] A retrofit, such as insulating both sides of each party wall, which reduces party wall heat transfer into attic to a minimal value.

TABLE 4

Heat loss rates (three-zone model)[a] (% of as-built townhouse)

House condition	Heat loss (%)[b]				Savings by retrofit (%)
	L → O[c]	A → O[c]	B → O[c]	Stack[d]	
As built	40	35	12	13	0
Plus ABCD retrofits [11]	40[e]	19	11	11	19
Plus hypothetical party wall retrofit	40[e]	4	11	8	37

[a] Assumes $T_L - T_O$ = 20 °C, I_{free} = 1.7 kW; includes stack losses.

[b] Direct heat loss to outside as defined by $W_{LO}(T_L - T_O)$, etc.

[c] "L → O" means living space to outside, "A → O" means attic to outside, "B → O" means basement to outside.

[d] Stack losses at exit from house.

[e] The retrofits are assumed not to affect this heat flow.

appear in eqn. (4) (the basement energy balance equation) it can be used to solve for W_{BA} in terms of known quantities. W_{LA} can then be determined from eqn. (5).

All the conductances thus obtained are shown in the first four columns of Table 3 for typical Twin Rivers townhouses as built and after various retrofits. Equation (8) may be used, along with these conductances, to determine the overall conductance (W_{net}) and furnace efficiency (ϵ) as defined by eqn. (7). The values of W_{net} indicate that considerable reduction in house "lossiness" can be made if an appropriate party wall retrofit is added to the ABCD or the super-retrofit packages. Modest increments in effective furnace efficiency (ϵ) are also predicted.

These conductances may also be used to calculate the heat loss rate from the house, attic and basement for any $T_L - T_O$. The heat loss rate I_{total} is given by:

$$I_{total} = I + I_{free} \tag{9}$$

and includes furnace inefficiency as a heat loss. Typical heat loss rates as a percentage of the value for an unretrofitted Twin Rivers townhouse are shown in Table 4 assuming $T_L - T_O$ = 20 °C and I_{free} = 1.7 kW (based on regression analysis of consumption data [14]. Perhaps the most interesting result is that 35% of the house heat loss occurs through the attic, prior to any retrofits. Table 4 also indicates the savings in heat loss rate — 19% and 37% of the "as-built" townhouse value after ABCD retrofits and ABCD-plus-party-wall retrofits. A more useful measure is savings in furnace input power, I, and turns out to be 23% and 45%, repsectively, when $T_L - T_O$ = 20 °C. The additional apparent savings occurs because I_{free} (assumed unchanged) is more effective in house heating after retrofits [13]. The gas savings of 23% following ABCD retrofits agrees well with measured values for a number of townhouses [14]. Gas savings following the party wall retrofit have not been verified yet.

It should be recalled that the model was developed using the steady state assumption. The steady state assumption implies that the model may lead to inaccurate predictions during the "edges" of the heating season. However, the furnace power during these periods is usually small.

Thus we see that a three-zone model for the house, including basement-to-attic and house-to-attic thermal bypass conductances and furnace inefficiencies, is consistent with measured attic and basement temperatures and predicts gas savings from retrofits accurately. Of course, we could not have made these predictions without first having obtained temperature and furnace data from a sample of houses. However, because of our experiments we are now in a position to make economically optimal suggestions for retrofitting the remaining 3000 Twin Rivers townhouses. Our final recommendations await a decision on the optimal way to block party wall heat transfer.

CONCLUSIONS

We have found that the warm-attic discrepancy in Twin Rivers attics implies that a three-zone house model is necessary for a complete understanding of thermal energy flow in these houses.

We have concluded that insulation bypass mechanisms are responsible for the loss of about 35% of the energy released in a Twin Rivers townhouse during cold months. Fortunately, these bypass mechanisms are correctable.

Furthermore, we have shown that the two-resistance model for attic temperatures is theoretically valid even if multiple zones are involved. This model can be used as a diagnostic tool to search out bypass mechanisms in different types of attics. If a discrepancy is found with handbook predictions, a multi-zone model (together with temperature data) may be used to quantify unsuspected heat loss mechanisms and to predict the effectiveness of any corrective action.

APPENDIX

Validity of the 2-resistance attic model in light of strong coupling to the house and basement

The large attic-to-basement heat transfer rate discussed in the text suggests that a simple two resistance circuit for the attic heat balance excluding the basement might be only a rough approximation. However, even an *n*-resistance circuit can be shown to reduce to an effective two-resistance attic model if the "free heat" component in the house is neglected. (If the free heat component is included in the circuit the deviations from the two-resistance model for the Twin Rivers houses should only amount to a few percent.)

To prove this result, consider a general *n*-element circuit which connects attic, basement, and house. The living space is considered to be at a uniform temperature, T_L, maintained constant by the thermostat setting and the furnace. Furnace heat plus internal sources of free heat are delivered inside the house. Indirect heating resulting from furnace inefficiencies are included as a fixed fraction of the furnace power added to the basement, attic or any other node in the circuit. A solution of this equivalent circuit for the inside–outside temperature difference, $T_L - T_O$, indicates that it is proportional to some linear combination of all the heat sources in the circuit. However, since all furnace-originated heat sources are proportional to the furnace power, we obtain:

$$T_L - T_O = aI + bI_{free} \tag{A1}$$

where I is the combustion power of the furnace, and I_{free} is the power delivered by applicances, people and the sun. The parameters a and b depend upon the resistances in the circuit and the fractions of furnace power deposited at the various nodes.

The furnace power (I) may be expressed as:

$$I = \frac{W_{net}\,(T_L - T_O) - I_{free}}{\epsilon} \tag{A2}$$

where W_{net} and ϵ depend on a and b.

We now make a similar analysis with $T_A - T_O$ (attic-to-outside temperature differences) as a variable. Since the network is linear, $T_A - T_O$ can only depend upon a linear combination of $T_L - T_O$ and the heat sources in the circuit.

Thus,

$$T_A - T_O = A(T_L - T_O) + BI + KI_{\text{free}}$$

where A, B and K are constants.

Substituting for I, from eqn. (A2), we obtain:

$$T_A - T_O = A + \frac{BW_{\text{net}}}{\epsilon}(T_L - T_O) + \left(K - \frac{B}{\epsilon}\right)I_{\text{free}} \qquad (A3)$$

Thus we see that when I_{free} is zero, $T_A - T_O$ is proportional to $T_L - T_O$, and eqn. (A3) is equivalent to a two-resistance attic model. A similar analysis can be made for the basement temperature, indicating that it too satisfied the proportionality requirement for a two-resistance model.

To examine the significance of the I_{free} term we have solved the three-zone circuit of Fig. 2. The coefficient of I_{free} turns out to be small enough so that inclusion of this term amounts to a small correction to the two-resistance model during the colder winter months. Of course, in the warmer months, at the "edges" of the heating season, the I_{free} term may not be negligible.

The key assumption in our argument involves the fixed proportionality between furnace combustion energy dumped into the various zones or nodes of the circuit. The furnace energy deposited in the living space and into the basement (by duct losses) must be roughly proportional to the furnace power. Additional heat added to the basement and attic by radiation from the furnace and the flue are also approximately proportional to the furnace power, provided the hot surfaces are much warmer than the ambient air, as is usually the case. Thus the assumption of constant fractions of furnace energy being deposited in the various nodes appears reasonable.

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