

Crack flow equations and scale effect

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1. INTRODUCTION

A major source of ventilation in many dwellings is that arising from air flow through cracks. To date, much use has been made (e.g. see references [1, 2]) of equations of the form

$$\Delta p = \text{Constant} \cdot V^n \quad (1)$$

for describing crack flows. In this equation Δp is the pressure drop across the crack, V is the volume flow rate through the crack and the exponent has a value of about 1.6. Equations of this type lack generality because they are not dimensionally homogenous. That is, they are in conflict with a fundamental law of fluid mechanics — Reynolds law of similitude.

From an earlier laboratory investigation of crack flows [1], improved semi-empirical equations have been derived. A method of applying them was proposed, but it has been found to have limitations and the present article is an attempt to provide an improved method.

Essentially, crack flow equations are required for two purposes.

- (i) For use in a prediction method for investigating the effects of dwelling configuration, mechanical systems and external wind on ventilation rates.
- (ii) For estimating the open areas of room components (e.g. doors, windows) when direct measurement is not possible.

The use of the equations in the above two ways and the implications of the equations with regard to the use of model-scale measurements of ventilation rates are discussed and some recent experimental results are presented.

2. DERIVATION OF EMPIRICAL EQUATIONS

Dimensional analysis indicates that for cracks with exact geometric similarity, the flow can be described by the functional relationship (see Appendix I)

$$C_z = f(R_{e_z}) \quad (2)$$

where C_z is the discharge coefficient

$$C_z \equiv \frac{V}{A} \sqrt{\frac{\rho}{2\Delta p}} \quad (3)$$

and the Reynolds number, R_{e_z} , is

$$R_{e_z} \equiv \frac{\bar{w} d_h}{\nu} \quad (4)$$

In the above equations, ρ and ν are the air density and kinematic viscosity respectively and \bar{w} is a mean velocity defined by

$$\bar{w} \equiv \frac{V}{A} \quad (5)$$

The symbol A denotes an area of the crack corresponding to some specified cross-section and d_h is a typical dimension defined by

$$d_h \equiv \frac{4A}{\text{wetted perimeter}} \quad (6)$$

For most cracks likely to be encountered in ventilation work

$$d_h = 2y \quad (7)$$

where y is the thickness of the crack as illustrated in Figure 1.

In practice, cracks take a variety of shapes and this implies a large number of distinct functions f . By introducing a geometric parameter one would hope to reduce this number to a manageable value. By analogy with pipe flows the ratio z/d_h suggests itself as a parameter, where z is the distance through the crack, as illustrated in Figure 1. Hence the crack flow equation is assumed to take the form

$$C_z = f\left(R_{e_z}, \frac{z}{d_h}\right) \quad (8)$$

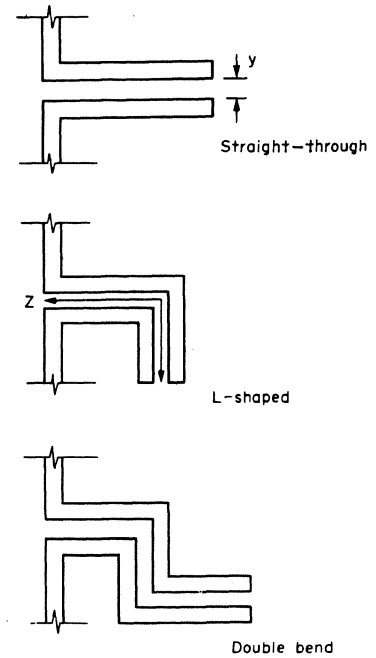


Figure 1

Common types of cracks encountered in buildings

The feasibility of using such an equation has been investigated [1] by carrying out tests on a large number of simulated cracks for ranges of R_{e_z} and z/d_h which are likely to be encountered in practice. The cracks were divided into three basic types — straight-through, L-shaped and double-bend (see Figure 1). For each type of crack it was found that the experimental results could be reasonably correlated by

$$\frac{1}{C_z^2} = B \frac{z}{d_h} \cdot \frac{1}{R_{e_z}} + C \quad (9)$$

Figures 2 (a—c) show the experimental results and the empirical values of the constants B and C chosen for the three crack types.

Equation (9) can be derived by equating the pressure drop across the crack to the sum of the losses due to skin friction and end effects (see Appendix II). In the above form, the skin friction contribution corresponds to laminar flow. The equivalent equation for turbulent flow is

$$\frac{1}{C_z^2} = B \frac{z}{d_h} \cdot \frac{1}{R_{e_z}^{0.25}} + C \quad (10)$$

and the results for the double-bend cracks are plotted in Figure 3 as $1/C_c^2 \sim z/d_c \cdot 1/R_c^{0.25}$. Comparing this with Figure 2 it is clear that equation (9) is more appropriate than equation (10) for correlating the experimental data.

In reference [1] the use of equation (9) in combination with equation (1) is proposed. However, the introduction of (1) is inconsistent and unnecessary, because the flow is completely and rigorously described by (9) alone and it is therefore now proposed that it should be used alone.

Undoubtedly, equation (9) is a simplified representation of a complex flow situation. Bearing in mind the wide variety of crack types which are met in practice, however, it seems that some degree of simplification is unavoidable. Indeed, it is uncommon to find a room component for which the crack type and the crack dimensions are everywhere constant.

Despite these problems it will be seen from Section 3.2 that equation (9) has been found to be successful both for estimating the open areas of real full-scale components and for describing the flow through them.

The advantages which equation (9) has relative to equation (1) are two-fold. Firstly, it is dimensionally homogenous so that the value of the constants do not vary with the particular system of units used. Secondly, it takes into account the effect of Reynolds number (i.e. scale effect) which is an important effect (see Sections 3.1 and 4). As a result of this, the estimated values of open area are independent of the flow rate through the open area. This also means that where an open area can in fact be obtained by direct measurement, the value can be used directly in the equation. Equation (9) is thus claimed to have wider validity than equation (1).

3. USE OF CRACK FLOW EQUATIONS

3.1 Prediction of ventilation rates

The crack flow equation (9) has been incorporated into a computer program for predicting the flow rates through cracks in a multi-cell dwelling. An iterative procedure is used to solve the crack flow and continuity equations.

Figure 4 shows some results obtained from this program in the form of a plot of the total air change rate of an actual nine-roomed house against reference wind speed, U_{ref} . The open areas of this house have been estimated experimentally as described in Section 3.2. For these calculations the pressure distribution over the house was estimated, but for future calculations a scale model has been constructed to enable more accurate distributions to be obtained in a wind tunnel. These calculations will be compared with ventilation measurements made in the house in order that the prediction method can be fully assessed.

Also shown in Figure 4 are the corresponding results obtained assuming that the discharge coefficient of the open areas is independent of Reynolds number and crack type. It can be seen that this simple square-law approximation gives significantly higher air change rates than the crack flow equations.

3.2 Indirect measurement of open areas

For the prediction of ventilation rates for a full-scale dwelling it is necessary to know the open areas of each room component.

In most cases a direct measurement of the full-scale area is not possible and an indirect method is employed. The pressure drop Δp across the component (at some point on the component) is measured, together with the corresponding volume flow rate. It can be shown that the flow equation can be written in the form of a simple cubic

$$A^3 \frac{2\Delta p}{\rho V^3} - C \cdot A - \frac{BzL^2}{4V} = 0 \quad (11)$$

which enables A to be calculated knowing Δp , V , the crack type (which determines B and C), z and L , where L is the length of the crack. Figures 5–7 show the solutions of this equation for the three types of cracks. Hence, knowing the above quantities from full-scale tests it is possible to obtain estimates of open areas, A .

In the following the results of tests on a door in the house which were carried out to verify the above procedure are described. The tests also constitute a check on the suitability of equation (9).

The tests were carried out in the small bedroom of the house. Prior to the tests, vinyl floor covering was laid and the room was extensively sealed with adhesive tape. An orifice plate was used to measure the extraction rate of the air which was exhausted through the vent of the installed warm-air system. The static pressure difference across the door was measured with a micromanometer.

The dimensions of the door are shown in Figure 8. It should be noted that the quoted dimensions of the crack thickness, z , are only approximate since there was some variation of this quantity over the door.

Three separate $V \sim \Delta p$ characteristics were recorded. Firstly, the characteristic corresponding to the whole door was obtained. The lower gap of the door was then sealed and the second characteristic recorded. Finally, the door was completely sealed and the base level characteristic was obtained.

The base level characteristic was used to correct the other characteristics. That is, at a given value of Δp , denote the base level extraction rate by V_{base} and the extraction rate for the test with the door unsealed by V_1 . The flow rate through the door at that value of Δp is then given approximately by $(V_1 - V_{base})$.

Figure 6 was then used to estimate the open area of the component from each measured point on the $V \sim \Delta p$ characteristic. These estimates are given in Figure 9. Also shown here are the estimates of A which are obtained from the $V \sim \Delta p$ characteristics on the assumption that C_c is constant ($C_c = 0.60$). It is clear that these latter estimates are far from satisfactory. The estimated open areas for the two door configurations are tabulated below, together with the values obtained from the direct measurement of A (see Figure 8).

	Area, A m ²	
	From crack flow equation	From direct measurement
Whole door	0.0157	0.0171
Lower gap sealed	0.0087	0.0095
Difference	0.0070	0.0076

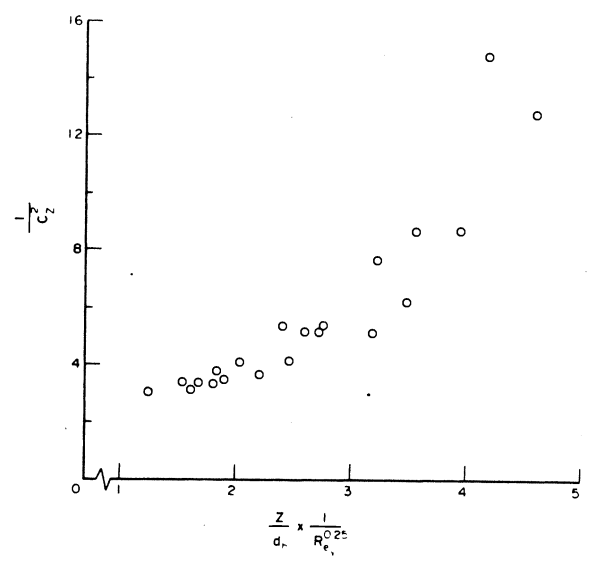
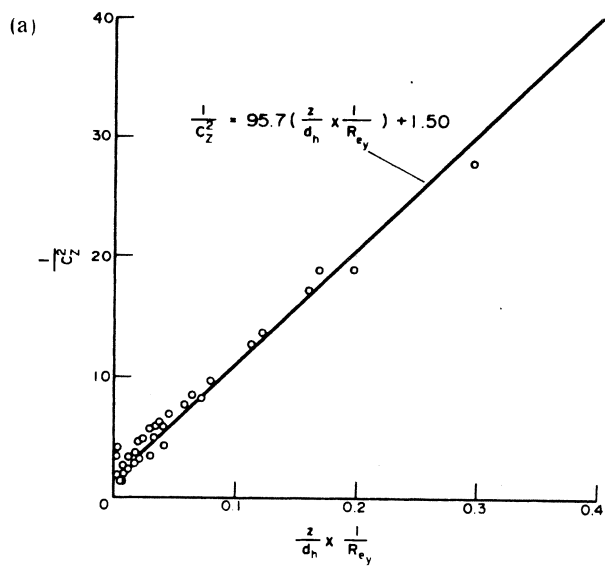


Figure 3
Experimental results for double-bend cracks plotted in accordance with turbulent flow

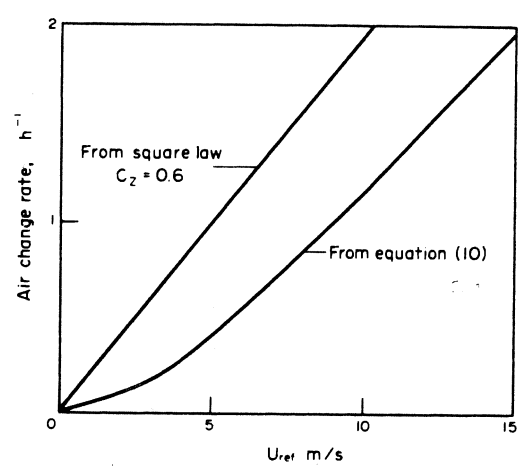
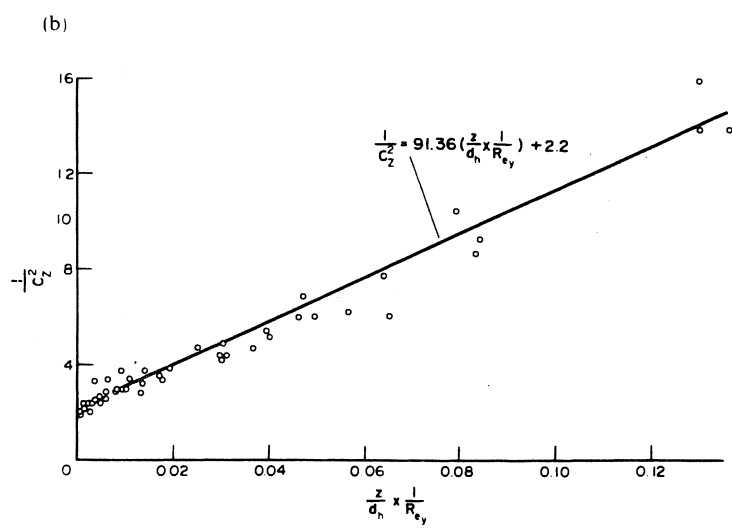


Figure 4
Calculated effect of wind speed on air change rate of a test house

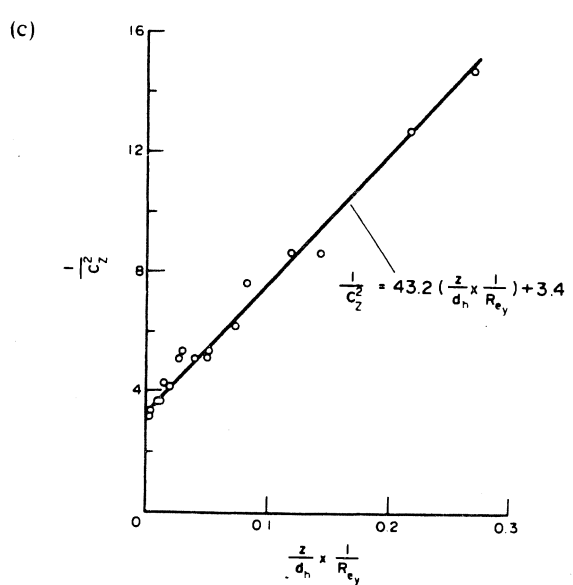


Figure 2
Experimental results and empirical lines chosen for the three crack types. (a) straight-through, (b) L-shaped, (c) double-bend

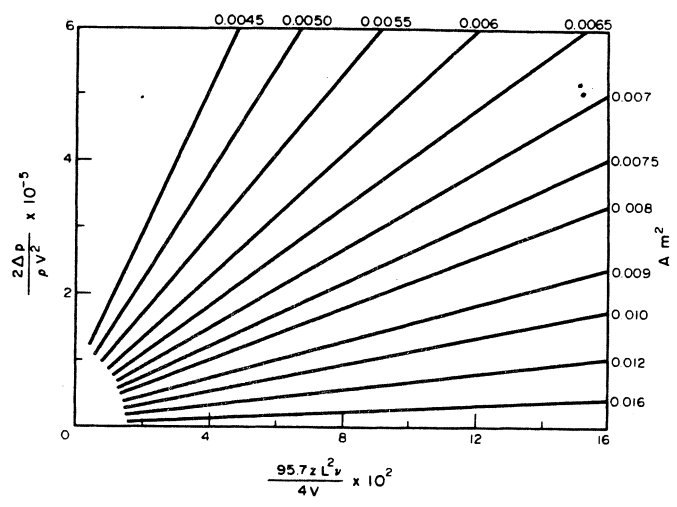


Figure 5
Solution of crack flow equation for straight-through cracks

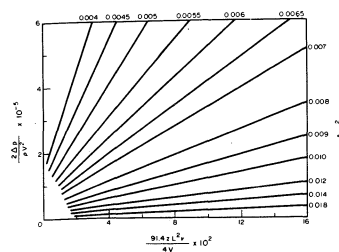


Figure 6
Solution of crack flow equation for L-shaped cracks

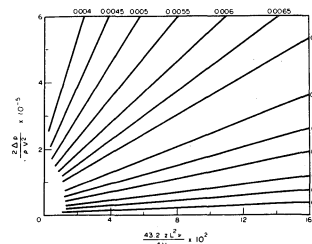


Figure 7
Solution of crack flow equation for double-bend cracks

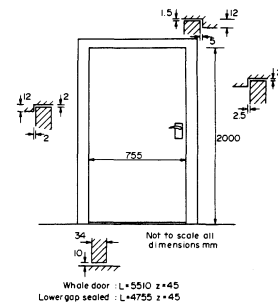


Figure 8
Dimensions of door in test house

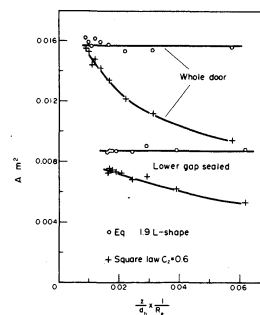


Figure 9
Estimated values of open area of door

It must be borne in mind when comparing the above values that it was not generally possible to measure γ directly with great accuracy because of variations over the door. The thickness of the lower gap did not vary appreciably, however, and thus the good agreement between the difference values is encouraging.

All of the doors and windows of the house have had their open areas estimated in the above manner and the results obtained are qualitatively similar to those given in Figure 9. This means that the flow through these components can be accurately calculated knowing the pressure difference across the components.

When using the above technique for obtaining leakage data of houses one is often faced with room components (doors, windows) for which the crack type varies. One therefore has the choice of either dividing each room component into smaller parts according to crack type, or treating the component as a whole. If the first choice is adopted, the open area of each part can be measured by the pressure/extract technique (by sealing the other parts) and the total open area is then obtained by summing the individual open areas. If it were desired to use the open areas for prediction of flow rates for the house, then each part of the component would be treated separately. For reasons of economy, however, it might be desirable to treat each room component as a whole. Tests on all of the components in the house have shown that it is permissible to do this. For example the results for the whole door given in Figure 9 show that the flow characteristics of the whole door are adequately described by the equation for L-shaped cracks. In general, the open area obtained in this way must be treated as an effective open area, since it will not generally be equal to the sum of the individual parts. However when the component consists of straight-through and L-shaped cracks (for which the flow equations do not differ greatly) with similar values of z , the results from the tests on the door indicate that the effective open area is approximately equal to the total open area.

An extreme example of the use of the flow equations for estimating open areas is given in Figure 10. Here the

'background' open area of a room has been estimated from the measured base level flow characteristics, $\Delta p \sim V_{b,0}^2$. For the measurements the doors and windows in the room were sealed, so that the flow occurred through the remaining cracks in the surfaces of the room. Since it was not possible to identify the geometry of these cracks, the values of L , B and C used for estimating open area were arbitrarily chosen. Nevertheless, the fact that the estimated values of background area do not vary greatly with flow rate, indicates that the crack flow equations can be usefully employed to describe very complex crack formations.

To summarise, it can be seen that the pressure/extract technique coupled with the crack flow equations given here, can be used to obtain open areas of room components. When applied to whole components with varying crack geometry, the open area obtained is an effective total open area, which may differ from the actual total open area. However this is of little consequence when the results are used as data for predicting flow rates, since the equations describe the flow characteristics of whole components very well. Even for background areas the equations give an adequate description of base level flows.

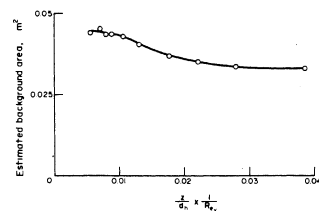


Figure 10
Estimated values of background area of a room

4. USE OF SCALE MODELS FOR DETERMINING VENTILATION RATES

Ventilation rates can be determined from model tests in wind tunnels either by measuring the external pressure distribution and using this as data for a theoretical prediction or by measurement of ventilation rates in the model. The use of a model for obtaining external pressure distributions for the purposes of design, development and/or research is an established procedure. The problem is whether or not the ventilation rates are best determined theoretically or by measurement at model scale.

A strong argument in favour of model scale measurement is that the available theory does not take account of the effects of wind turbulence and internal air movements. Both of these phenomena are at least present in model tests and there is evidence (e.g. references [3, 4]) that the former effect can be very significant.

There are however quite strong arguments against the use of model scale measurement. Firstly, it is difficult to model full-scale cracks accurately unless these are large or the model scale is large (1/25 scale, say). Secondly, it is generally not possible to achieve full-scale Reynolds number at model scale

because of limitations on tunnel speed. Since the flow through cracks tends to be laminar, Reynolds number effects are likely to be significant. Figure 11 shows the theoretical variation of flow rate, $V/(U_{ref}A)$, through typical windows with reference wind speed U_{ref} . It can be seen that even with a large crack thickness of 5 mm, Reynolds number effects are significant up to a speed of about 5 m/s. To achieve the same Reynolds number with a 1/25 scale model would require tunnel speeds up to 125 m/s. For the smaller crack sizes, wind tunnel simulation over the required Reynolds number range appears to be even less feasible.

The relative importance of the effects of Reynolds number and turbulence has been investigated experimentally in a wind tunnel of the type described in reference [6]. A grid of horizontal slats was used to generate the simulated atmospheric boundary layer. Figure 12 shows the resultant profiles of mean velocity, U/U_{ref} , and turbulence intensity, U_{rms}/U_{ref} . These measurements were made with a pitot-static tube which was considered adequate for the present purposes. The intention was to obtain profiles appropriate to an open-country site, and the corresponding power law for the mean velocity is shown for comparison. A single-cell model fitted with identical model windows on two opposite faces, as shown in Figure 13, was chosen for the tests. Ventilation rates were obtained from tracer gas decay records measured by a katharometer installed inside the model. A full report on the tests will be published at a later date, but some relevant results are presented in Figure 14. The measured variation of $V/(U_{ref}A)$ with U_{ref} is shown for two wind directions $\alpha = 0^\circ$ and 90° . At $\alpha = 0^\circ$ the mean pressure difference across the windows is at a maximum, whereas at $\alpha = 90^\circ$ the mean pressure difference is negligibly small and the ventilation rate is due to turbulence effects alone. The U_{ref} axis can be considered as a Reynolds number axis.

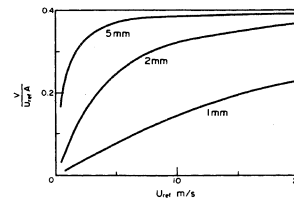


Figure 11
Theoretical calculations of scale effect for typical full-scale crack sizes

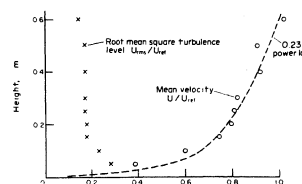


Figure 12
Profiles of mean velocity and turbulence intensity in the wind tunnel

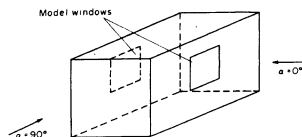


Figure 13
Wind-tunnel model used for measurement of ventilation rates through model windows and circular holes

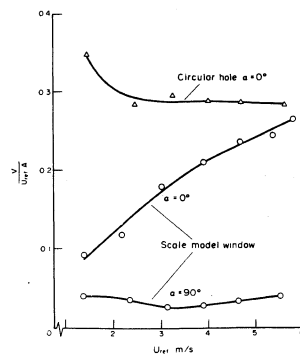


Figure 14
Scale effects for model windows and circular holes as measured with the wind-tunnel model

It can be seen that the ventilation rate due to turbulence alone ($\alpha = 90^\circ$), when suitably non-dimensionalised, is roughly independent of Reynolds number. In contrast the dimensionless ventilation rate for $\alpha = 0^\circ$ increases rapidly with Reynolds number i.e. scale effect is significant. As far as magnitudes are concerned, at the highest Reynolds numbers tested, the ventilation rates at $\alpha = 0^\circ$ are much larger than those at $\alpha = 90^\circ$. However, at the lowest Reynolds numbers the ventilation rate due to turbulence alone is appreciable in relation to the ventilation rate for $\alpha = 0^\circ$.

Also shown in the figure are the results obtained for $\alpha = 0^\circ$ when the model windows are replaced by circular holes, as used by other workers (e.g. reference [5]). The flow characteristics of the two types of open area are clearly different, with the non-dimensional ventilation rate of the circular holes tending to a constant value at a relatively low Reynolds number. This behaviour indicates that circular holes can be used to model the behaviour of real windows in the limit of high Reynolds number. The open area of the circular holes should not be simply scaled geometrically, however, but allowance should be made for the different values of the discharge coefficients of full-scale cracks and of circular holes.

5. CONCLUSIONS

Crack flow equations of the form of equation (1) are not satisfactory because they do not satisfy Reynolds law of similitude. Earlier investigations (reference [1]) have shown that the flow through simulated full-scale cracks can be adequately described by the semi-empirical equation (9). It is now confirmed that this equation should not be used in the manner described in reference [1], but should be used on its own for obtaining the discharge coefficient.

Full scale tests on a door in a house have been found to support the use of equation (9) for estimating open areas of real full-scale room components. It has also been found from tests of all the room components that the equation satisfactorily describes the flow through a wide range of components.

The usefulness of ventilation rates measured at model scale for the design of full-scale dwellings is open to doubt, because of the large effect of scale implied by the crack flow equations. Calculations indicate that scale effect is apparent even at the Reynolds numbers associated with full-scale dwellings.

Tests carried out on a model in a wind tunnel have illustrated the significance of scale effect when the ventilation rate is due mainly to the existence of a time-mean pressure difference across a crack. When the time-mean pressure difference is zero, and the ventilation rate is due solely to turbulent pressure fluctuations, the scale effect is much smaller. * At the lowest Reynolds numbers tested, the ventilation rate due to turbulence alone is about one half of the ventilation rate which exists when the time-mean pressure difference is maximised.*

The model tests have also illustrated the different flow characteristics of scale model cracks and circular holes. Circular holes can be used to model the behaviour of real cracks in the limit of high Reynolds number, provided that the open areas of the holes are correctly determined.

ACKNOWLEDGEMENTS

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*See *Build. and Env.*, 14, 53-64 (1979) for further discussion of this conclusion.

APPENDIX I

The dimensionless groups which describe the flow through geometrically similar pipes can be obtained by dimensional analysis in the manner shown for example in reference [7]. One obtains

$$\frac{\Delta p}{\rho \bar{w}^2} = f_1 \left(\frac{\bar{w} \cdot D}{\nu} \right) = f_1(R_{ep}),$$

where D is the pipe diameter and R_{ep} is the corresponding Reynolds number. The analysis for flow through geometrically similar cracks is the same, and one obtains

$$\frac{\Delta p}{\rho \bar{w}^2} = f_1 \left(\frac{\bar{w} \cdot d_h}{\nu} \right) = f_1(R_{ep}),$$

which, on putting $\bar{w} = V/A$, can be expressed in a form involving the discharge coefficient, i.e.:

$$\frac{1}{C_d^2} = f_1(R_{ep})$$

or

$$C_d = j(R_{ep}),$$

APPENDIX II

The total pressure drop across the crack can be considered as the sum of pressure drops due to skin friction (of the form encountered in long straight cracks) and due to bends and end effects. This is the procedure which is generally adopted for pipe flows (e.g. see reference [7]). It is assumed that the pressure drops caused by bends and end effects are simply proportional to $\rho \bar{w}^2/2$.

In the following, equation (9) is derived using the above procedure, in a similar way to that given in reference [1].

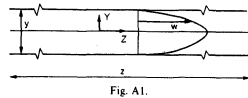


Fig. A1.

For steady laminar flow between parallel walls, the velocity distribution is parabolic (see e.g. reference [8]), i.e.:

$$w(Y) = \frac{1}{2\mu} \frac{dp}{dz} \left(\left(\frac{y}{2} \right)^2 - Y^2 \right) \quad (A1)$$

In the present notation Y and Z denote distance in the crosswise and streamwise directions, respectively, with the origin of Y at the centre-line. The crack has breadth y and length z . $w(Y)$ denotes the streamwise velocity at Y and since the flow is parallel everywhere it follows that the pressure gradient is constant, so one can put

$$\frac{dp}{dz} = \frac{\Delta p}{z}.$$

From (A2), the mean velocity \bar{w} , defined by equation (5) is given by

$$\bar{w} = \frac{V'}{y} = \frac{1}{y} \int_{-y/2}^{y/2} w(Y) \cdot dY,$$

where V' is the volume flow rate per unit width.

On integration one obtains

$$\bar{w} = \frac{1}{12} \frac{\Delta p \cdot y^2}{\mu z}, \quad (A3)$$

Now

$$\frac{1}{C_d^2} = \frac{2\Delta p}{\rho \bar{w}^2}$$

and since

$$R_{ep} = \frac{\bar{w} d_h}{\nu} \quad \text{and} \quad d_h = 2y$$

we can express (A3) in the form

$$\frac{1}{C_d^2} = 96 \cdot \frac{1}{R_{ep}} \cdot \frac{z}{d_h} \quad (A4)$$

Proceeding in the manner described above, the total pressure drop, Δp_T , is given by

$$\Delta p_T = \frac{\rho \bar{w}^2}{2} \cdot 96 \cdot \frac{1}{R_{ep}} \cdot \frac{z}{d_h} + \frac{\rho \bar{w}^2}{2} \cdot C$$

where C is a constant. Thus the flow through the crack is described by

$$\frac{1}{C_d^2} = 96 \cdot \frac{1}{R_{ep}} \cdot \frac{z}{d_h} + C$$

which is the required form.

It is interesting to note that the empirical values of B obtained for the straight-through and L-shaped cracks are close to the value 96. This supports the assumption that the pressure drop can be considered as two additive components. The value of B obtained for the double-bend crack type, indicates that the assumption becomes less tenable as the number of bends is increased. Nevertheless the flow is still adequately described by an equation of the same form.