Wind, Temperature and Natural Ventilation - Theoretical Considerations*

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The weather drives air infiltration by two separate physical mechanisms: (1) wind and (2) convection induced by a temperature difference between indoors and outdoors. These two mechanisms have complex interactions that are sensitive to the location of cracks in a building. The nature of the interaction of the two effects is displayed pictorially for several idealized examples. In an Appendix, the subadditivity of the effects for a wide class of situations is proven mathematically: the combined effect of wind and temperature is never greater than one would estimate by simple addition of the independent effects.

In temperate zone houses, air leakage through cracks and crevices typically accounts for a third or more of the winter heat load and a somewhat smaller fraction of the summer air-conditioning load. How this leakage is linked to the weather is the subject of this article.

The weather drives the leakage flow by two separate physical mechanisms: (1) wind, and (2) temperature-induced convection. Unfortunately, these do not act independently; *i.e.* their effects cannot be simply superimposed. Rather, they interact in a complex way that depends on the pattern in which cracks and crevices happen to be distributed over the surface of the house. For some patterns, the effects of wind and temperature tend to cancel; for other patterns, they tend to add. In fact, for a given fixed pattern of cracks and crevices, the nature of the interaction may vary with the wind and temperature themselves, cancelling in some ranges and adding in others. Simple examples given below illustrate these effects.

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The complexity of the wind–temperature interaction, though discussed here in theoretical terms, has important implications for practice. It is, for example, bad news for computer modelers since it appears unlikely that there exists any simple general formula that universally represents natural ventilation in buildings. This conclusion is reinforced by field observations. Linear regressions of measured air infiltration against wind and temperature do in fact show erratic results; sometimes the fit is good and sometimes it is not [1]. This is exactly what one would expect on the basis of the considerations given below.

Fortunately, achieving the goal of saving energy by rationalizing ventilation does not depend crucially on having a satisfactory computer model. An alternative approach is to develop a practical set of techniques and instruments that can be used in the field to obtain the information necessary to specify and test conservation measures for particular buildings.

Let:

A = rate of air infiltration for a particular building (m^3/s) (equals rate of air exfiltration),

W = wind velocity (m/s),

ΔT = inside minus outside temperature (°C).

If the effects of wind and temperature were additive, then the function $A(W,\Delta T)$ could be written in the form:

$$A(W,\Delta T) = A(W,0) + A(0,\Delta T) \qquad (1)$$

It will be shown below that this, in general, is not possible. Failing this, one might hope that $A(W,\Delta T)$ would at least be monotonic, *i.e.* that an increase in either wind W or temperature difference ΔT would always result in increased air flow:

$$\frac{\partial A}{\partial W} > 0, \; \frac{\partial A}{\partial \Delta T} > 0 \text{ for all } W, \Delta T \qquad (2)$$

But it will be shown that even this is not true in general.

It *is* true, however, that under quite general conditions $A(W,\Delta T)$ is *subadditive*:

$$A(W,\Delta T) \leqslant A(W,0) + A(0,\Delta T) \qquad (3)$$

This means that the combined effect of wind and temperature is never greater than one would estimate by simple addition of the independent effects. (The proof of this inequality is presented in the Appendix.)

The first section of this article reviews briefly the flow of air through a simple opening. The second section, through a series of simple examples, attempts to give an intuitive picture of the interaction between wind and temperature-induced convection, and in particular to make plausible the results cited above. The last section gives a general mathematical formulation and a formal proof of the subadditivity of $A(W,\Delta T)$.

FLOW THROUGH A SINGLE CRACK

Let p_0, p_1 = pressure outside and inside respectively, as in Fig. 1.

$$\Delta p = p_0 - p_1$$

The air flow A as a function of the pressure difference Δp can generally be approximated by:

$$A = K(\Delta p)^{\alpha} \qquad (4)$$

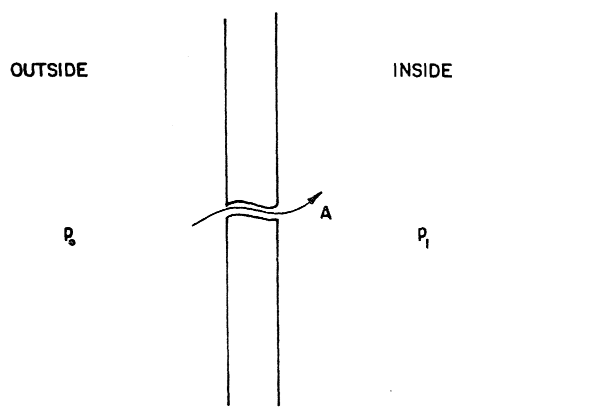

Fig. 1. Pressure difference drives flow through a crack.

where the constants K and α are determined by the shape and size of the opening. The exponent α varies with the flow regime as follows [2]:

laminar:	$\alpha = 1$
turbulent:	$\alpha = 4/7$
entrance, exit effects	$\alpha = 1/2$

Unfortunately, the dimensions and velocities found under ordinary circumstances are such that any or all of the three regimes may occur. The most that can be said in general is that $A(\Delta p)$ is a concave function that can be approximated by the form given above with some compromise α between 1/2 and 1.

SIMPLE EXAMPLES OF WIND–TEMPERATURE INTERACTION

Consider first a building (for simplicity a rectangular box) that is tightly sealed, and let $\Delta p(z)$ be the outside–inside pressure difference at the point z on the building's shell. We consider in this section what the pressure difference distribution $\Delta p(z)$ looks like under various conditions.

Suppose, for example, that the air is less dense inside than outside, as would be true for a heated building on a cold day. Then as the point z moves downward, the static pressure increases less rapidly inside than outside, and the pressure difference Δp therefore increases. If we arbitrarily assume that Δp is zero at mid-height, then the distribution of Δp over the vertical walls is shown in Fig. 2.

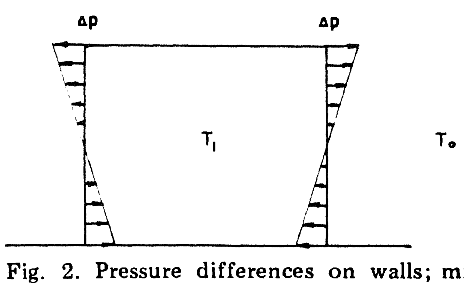

Fig. 2. Pressure differences on walls; mild day, no wind.

If the density difference is greater (due, *e.g.*, to a greater temperature difference) then the pressure difference variation is more pronounced as shown in Fig. 3.

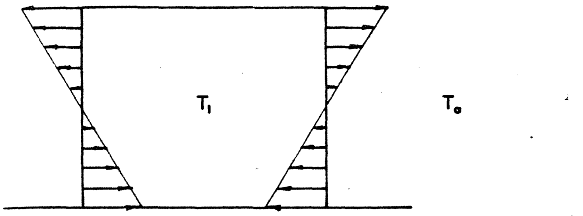

Fig. 3. Pressure difference on walls; cold day, no wind.

Now suppose that a wind is blowing from left to right, and for simplicity suppose that the effect of the wind is to increase the pressure uniformly on the windward side and to decrease it uniformly by a like amount on the leeward side. Then the distribution of Δp on the vertical side is shifted as shown in Fig. 4.

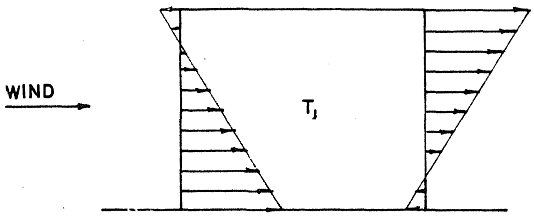

Fig. 4. Pressure difference on walls; cold day, slight wind.

If the wind is blowing harder, then the distribution is shifted further, as shown in Fig. 5.

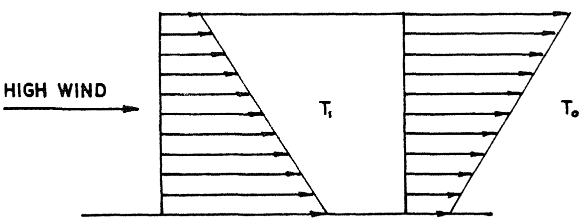

Fig. 5. Pressure difference on walls; cold day, high wind.

Since the density difference is very nearly proportional to the temperature difference,* Figs. 2 - 5 show essentially how the pressure difference distribution $\Delta p(z)$ varies with wind W and temperature difference ΔT under the simple assumptions we have made.

Suppose now that the sides of the building are not impervious, but rather are uniformly porous, and for the sake of simplicity of exposition, suppose that the pores are such that the coefficient α in eqn. (4) is equal to one. Then the rate of air flow through the wall is just proportional to Δp and the arrows in Figs. 2 - 5 can be reinterpreted as air flow vectors. The Figures have been drawn so that the total inflow equals total outflow. The inside pressure always adjusts itself so that this is the case.

The infiltration rate is obtained by summing the inward flow vectors. Representing this sum by A, one can see by inspection of the Figures that A is not a separable function of wind W and temperature difference ΔT as expressed by eqn. (1). In Fig. 5, for example, one can see that changing ΔT slightly has no effect whatever on A, since it changes only the slant of the distribution of inward arrow lengths, but not their sum. In Fig. 2, however, changing ΔT (the slant of the distribution) has a definite effect on A. Thus the effect of temperature difference is not independent of wind.

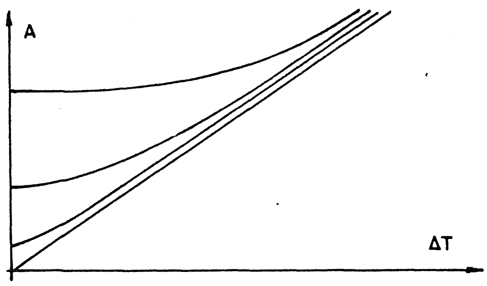

Fig. 6. A as a function of ΔT for various fixed wind velocities in the case of uniformly porous walls.

Figure 6 shows a plot of A *versus* ΔT for various wind speeds. The interdependence is clear. The case shown in Fig. 6, however, is relatively benign. A more striking case is shown in Fig. 7. Here, the uniformly porous

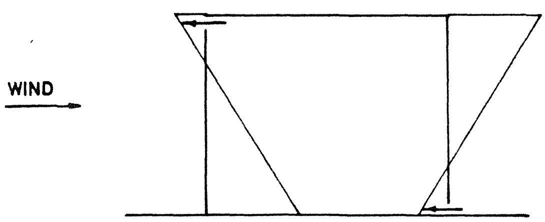

Fig. 7. With openings arranged as shown, the wind and temperature effects tend to cancel.

*The density difference is exactly proportional to $(1/T_0) - (1/T_1)$ where T_0 and T_1 are the absolute outside and inside temperatures, but this expression is very nearly proportional to ΔT.

walls have been replaced by solid walls with just two small openings — one at the top of the windward wall and the other at the bottom of the leeward wall. As the wind increases from zero, the air infiltration rate actually decreases as the wind progressively cancels out more and more of the temperature-induced flow. At some wind velocity, the effects cancel totally, and for higher wind velocities the flow reverses and increases in magnitude with increasing wind. Plots of A *versus* W for various fixed temperature differences are shown in Fig. 8. This is a case in which $A(W,\Delta T)$ is not only inseparable, but is not even monotonic.

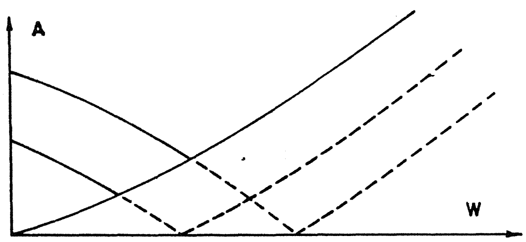

Fig. 8. A as a function of W for various fixed temperature differences when the openings are located as shown in Fig. 7.

Of course, these examples do not prove that $A(W,\Delta T)$ is *always* inseparable or non-monotonic, and indeed this is not true.

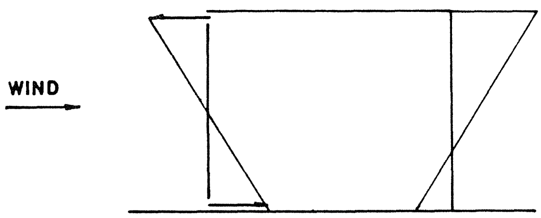

Fig. 9. Infiltration rate A is independent of wind under the simple assumptions given in text.

Figure 9 shows an example in which (under our simple assumptions) the infiltration rate A is entirely independent of wind, and Fig. 10 shows a companion example in which A is entirely independent of temperature difference. In these cases $A(W,\Delta T)$ is both separable (trivially) and monotonic.

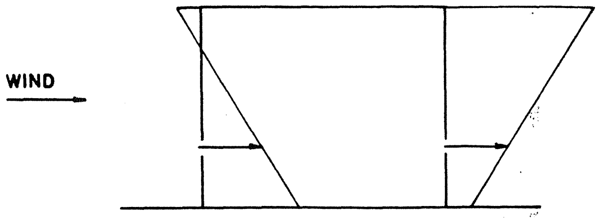

Fig. 10. Infiltration rate A is independent of temperature difference under the simple assumptions given in text.

These examples, of course, are based on simple assumptions. Real air flow is much more complex. In practice the function $A(W,\Delta T)$ is affected not only by the location of openings but also by the aerodynamic idiosyncrasies of individual buildings and by irregularities in the flow itself. Atmospheric turbulence, for example, can cause appreciable air exchange through openings even at low wind speed, so that $A(0,0)$ may not really be zero as assumed above. (See ref. [3] for a study of this effect).

In conclusion, then, it is not surprising that the air infiltration response of different individual buildings to wind and temperature is often puzzling and seemingly erratic.

REFERENCES

1 See, for example, N. Malik, Field investigations of air infiltration in homes, *Energy and Buildings*, 1 (1977/78) 281.
2 See, for example, F. J. Bailey, *An Introduction to Fluid Dynamics*, Interscience, New York, 1958, pp. 89, 101, 105.
3 J. E. Hill and T. Kusuda, Dynamic characteristics of air infiltration, *ASHRAE Trans.*, 81 (Part I), (1975)

APPENDIX

General characteristics of the function $A(W,\Delta T)$

In the absence of information about a building's details it would seem that little could be said about the function $A(W,\Delta T)$. We know that it is not necessarily linear or even separable or monotonic. Nevertheless it does have one useful property under quite general conditions, namely, the property of subadditivity as expressed in Inequality (3). This is stated and proved more precisely below.

Let:

z = a location on the building's shell,

$a(z,\Delta p)$ = air flow through the shell at point z in response to the local pressure difference, Δp.

The function $a(z,\Delta p)$ will be assumed to have the following simple properties:

(1) $a(z,\Delta p)$ always has the same sign as Δp and $a(z,0) = 0$,

(2) $a(z,\Delta p)$ is monotonic with respect to Δp: $\Delta p_1 > \Delta p_2$ implies that $a(z,\Delta p_1) > a(z,\Delta p_2)$,

(3) $a(z,\Delta p)$ is subadditive* with respect to Δp:

$$a(z,\Delta p_1 + \Delta p_2) \leqslant a(z,\Delta p_1) + a(z,\Delta p_2)$$

Note that the special form given in eqn. (4) has all of these properties.

Let:

$\Delta p_W(W,z)$ = pressure difference at point z due to wind W when $\Delta T = 0$,

$\Delta p_T(\Delta T,z)$ = pressure difference at point z due to temperature difference ΔT when $W = 0$.

The overall pressure difference when both W and ΔT are acting is:

$$\Delta p = \Delta p_W + \Delta p_T + \Delta p_0 \tag{5}$$

where Δp_0 is a constant which adjusts itself so that total flow in equals total flow out, *i.e.* so that

$$\int_S a(z,\Delta p)\mathrm{d}z = 0, \qquad S\text{: whole surface}$$

For fixed W, ΔT consider the subsets of S defined as follows:

*In particular, any concave function is subadditive.

S_W = set of z such that $\Delta p_W \geqslant 0$

S_T = set of z such that $\Delta p_T \geqslant 0$

$S_{\Delta p}$ = set of z such that $\Delta p \geqslant 0$

A bar over a set designation will indicate the complement, *i.e.*, $\bar{S}_W$ is the set of z such that $\Delta p_W < 0$.

The infiltration rate with both W and ΔT acting is:

$$A(W,\Delta T) = \int_{S_{\Delta p}} a(z,\Delta p)\mathrm{d}z \tag{6}$$

With wind acting alone the infiltration rate is:

$$A(W,0) = \int_{S_W} a(z,\Delta p_W)\mathrm{d}z \tag{7}$$

and with temperature acting alone it is:

$$A(0,\Delta T) = \int_{S_T} a(z,\Delta p_T)\mathrm{d}z \tag{8}$$

Theorem: If $a(z,\Delta p)$ has the three properties listed above, then

$$A(W,\Delta T) \leqslant A(W,0) + A(0,\Delta T)$$

Proof: For a fixed $W,\Delta T$ suppose that the constant Δp_0, which equates inward and outward flow, is non-positive: $\Delta p_0 \leqslant 0$. It follows from eqn. (5) and the definitions of the subsets of S, that:

$$S_{\Delta p} \subset S_W \cap S_T + S_W \cap \bar{S}_T + \bar{S}_W \cap S_T$$

Hence:

$$S_{\Delta p} = S_{\Delta p} \cap S_W \cap S_T + S_{\Delta p} \cap S_W \cap \bar{S}_T + S_{\Delta p} \cap \bar{S}_W \cap S_T \tag{9}$$

To demonstrate the inequality of the theorem, the procedure is to partition the domain of the integral in eqn. (6) into three subdomains according to eqn. (9), then to apply the properties (1), (2), (3) of $a(z,\Delta p)$ appropriately to the three integrals, and finally to expand the domains. This generates a chain of inequalities linking the two sides of the inequality to be proved. The details of the procedure are displayed in diagrammatic form below. The inequality signs link expressions above and below, reading downward. Where a change is only in an integrand with the domain held fixed, the integral sign is omitted to avoid cluttering. The encircled numbers next to the inequality signs refer to notes below which justify the steps.

$$A(W,\Delta T) = \int_{S_{\Delta p}} a(z,\Delta p)\mathrm{d}z$$

$$= \int_{S_{\Delta p}\cap S_W\cap S_T} a(z,\Delta p_W + \Delta p_T + \Delta p_0)\mathrm{d}z \;+\; \int_{S_{\Delta p}\cap S_W\cap \bar{S}_T} a(z,\Delta p_W + \Delta p_T + \Delta p_0)\mathrm{d}z \;+\; \int_{S_{\Delta p}\cap \bar{S}_W\cap S_T} a(z,\Delta p_W + \Delta p_T + \Delta p_0)\mathrm{d}z$$

$$\begin{array}{ccccc}
\leqslant (1) & & \leqslant (1) & & \leqslant (1) \\
a(z,\Delta p_W + \Delta p_T) & & a(z,\Delta p_W) & & a(z,\Delta p_T) \\
\leqslant (2) & & \downarrow & & \downarrow \\
a(z,\Delta p_W) + a(z,\Delta p_T) & & & & \\
\leqslant (3) & \downarrow & \leqslant (3) & & \\
A(W,0) = \int_{S_W\cap S_T} a(z,\Delta p_W)\mathrm{d}z & + & \int_{S_W\cap \bar{S}_T} a(z,\Delta p_W)\mathrm{d}z & & \downarrow \\
 & \leqslant (3) & & & \leqslant (3) \\
A(0,\Delta T) = & \int_{S_W\cap S_T} a(z,\Delta p_T)\mathrm{d}z & & + & \int_{\bar{S}_W\cap S_T} a(z,\Delta p_T)\mathrm{d}z
\end{array}$$

$$\therefore A(W,\Delta T) \leqslant A(W,0) + A(0,\Delta T)$$

(1) By monotonicity of $a(z,\Delta p)$. Terms in Δp which are negative over the domains are dropped. This cannot decrease the integrand.

(2) By subadditivity of $a(z,\Delta p)$.

(3) In each case the domain of integration is expanded by deleting the symbols $S_{\Delta p}\cap$. Since the integrand in each case is non-negative over the new domain, this cannot decrease the integral.

The proof to this point depends on the assumption $\Delta p_0 \leqslant 0$. Under this assumption it was shown that the *infiltration* rates satisfy the inequality of the theorem. It follows *ex post facto* that the exfiltration rates in absolute value also satisfy the inequality, since these are the same. If $\Delta p_0 \geqslant 0$, then the proof can simply be turned around, so that it applies directly to exfiltration. Thus the complements $\bar{S}_W$, $\bar{S}_T$, $\bar{S}_{\Delta p}$ replace S_W, S_T, $S_{\Delta p}$ and the sign of $a(z,\Delta p)$ is changed (since the inequality to be proved holds for the absolute value of exfiltration). Thus transformed, the proof goes through as before.

The last point perhaps becomes clearer when one reflects that the symmetry between outside and inside is perfect. There is nothing in the model that allows one to tell which is which except by arbitrary assertion. The difference between $\Delta p_0 \leqslant 0$ and $\Delta p_0 \geqslant 0$ is simply that in one case, equalization of flow is achieved by adding a constant pressure to the outside. Since outside is indistinguishable from inside, the cases are really the same.